Modelling the Dependence Structure of MUR/USD and MUR/INR Exchange Rates using Copula

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ABSTRACT: American Dollar (USD) and Indian Rupee (INR) play an important role in Mauritian economy. It is important to model the pattern of dependence in their co-movement with respect to Mauritian Rupee (MUR), as this may indicate the export-import behavior in Mauritius. However, it is known that distributions of exchange rates are usually non-normal and the use of linear correlation as a dependence measure is inappropriate. Moreover it is quite difficult to obtain the joint distribution of such random variables in order to specify the complete covariance matrix to measure their dependence structure. In this paper, we first identify the marginal distributions of the exchange rates of MUR against USD and INR and then select the best fitting copula model for the bivariate series. It is concluded that both the series are asymmetric and fat-tailed following hyperbolic distribution. Their dependence structure is appropriately modeled by t copula.

Keywords: Bivariate copula; Dependence structure; Exchange rates; Hyperbolic Distribution
JEL Classifications: C2; F

1. Introduction

Mauritius has several Indian and American trade transactions and treaties. Therefore, in Mauritian economy, both the Indian Rupee (INR) and the American Dollar (USD) play significant roles in connection with Mauritian Rupee (MUR). Since, INR and USD are interrelated due to global trade, the exchange rates MUR/USD and MUR/INR are expected to be interdependent. It is of interest to model this dependence in order to explore the joint movement of MUR against USD and INR. This would help to understand the import and export business pattern in Mauritius.

Joint probability density function is required to measure the dependence between the variables but it is quite difficult to derive joint densities in most of the cases. However, if the joint density is Gaussian, the correlation may be used as a measure of dependence but when the joint density is unknown or it is known to be non-Gaussian, the correlation fails to explain the dependence structure. Many studies such as McFarland et al., (1982), Chinn and Frankel (1994), Ausloos and Ivanova (2001), and Brunnermeier et al. (2009) have shown that the distributions of several exchange rates are either fat-tailed, skewed or leptokurtic. It is therefore obvious that normal distribution will provide a bad fit to exchange rate series. The student and mixture of two normals were chosen by Booth and Glassman (1987) as best fitting distributions for daily changes in the logarithms of exchange rates. Tucker and Pond (1988) observed that exchange rates of US dollar against six currencies: British pound, Canadian dollar, Deutschemark, French franc, Japanese yen and Swiss franc exhibit non-Gaussian characteristics and suggested to use mixed-jump model based on Poisson and normal distribution. Inspite of concrete evidence about the non-Gaussian nature of the daily exchange rates as well as logarithms of exchange rates, researchers have been assuming that the marginal distributions are Gaussian [Coppes (1995)]. Not only this, there is abundant literature which further assumes that joint distribution of exchange rates is also multivariate normal and employ correlation to measure the

dependence structure. This assumption is not realistic (Patton (2006), Dias and Embrechts (2009)) and mainly used because construction of exact joint density functions is not easy.

Copula approach has been able to bypass the need of complete covariance matrix to be derived from joint density function in order to explain the dependence structure. It provides a copula function that can link any type of marginal distributions to the joint distribution and analyses the marginal distributions separately from the dependence structure. The basic concept of copulas was given by Sklar (1959). Detailed explanation on copulas can be found in Patton (2009). Copula methodology is widely used in financial risk management [Aas (2004), Demarta and Mcneil (2005), Cherubini et al (2004). Bivariate copulas have been used to model the dependence of exchange rate of a currency against two currencies. These include USD, GBP, EUR, YEN, FRANC, PLN, CZK, Latin American currencies and many more. However, no models exist for the exchange rate of MUR against USD and INR in the literature. In section 2, brief description of copula models is provided. In section 3, we identify the best fitting distributions to MUR/USD and MUR/INR exchange rates. In section 4, we obtain the copula model which can best describe the dependence structure between the two exchange rates. The paper concludes in section 5.

2. Copula Models

Let $G(x,y)$ be the bivariate cumulative distribution function and $F_X(x)$, $F_Y(y)$ be the marginal cumulative distribution functions of random variables X and Y respectively. Then copula C is a function such that

$$G(x,y) = C(F_X(x), F_Y(y)) = C(u,v).$$

The lower limit of copula function is given by $C^-(u,v) = \max(u+v-1; 0)$ indicating the total negative dependence and the upper limit is given by $C^+(u,v) = \min(u,v)$ indicating total positive dependence. Two widely used families of copulas are:

- Elliptical copulas including Gaussian copula and t copula.
- Archimedean copulas including Clayton copula, Frank copula and Gumbel copula among others.

Elliptical copulas are suitable for modeling the dependence structure in symmetric data. Gaussian copula cannot model the tail dependence whereas t copula can model it. Clayton copula exhibits left-tailed asymmetry and Gumbel copula exhibits right-tailed asymmetry. Frank copula is the only symmetric Archimedean copula.

The Gaussian copula is defined as:

$$C^{Ga}(u, v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$$

(1)

where $\Phi_\rho(..)$ is the joint distribution function of a bi-dimensional standard normal vector with the linear Pearson correlation coefficient ρ. Thus,

$$\Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\left(\sqrt{1-\rho^2}\right)} \exp\left(\frac{2\rho st - s^2 - t^2}{2(1-\rho^2)}\right) ds\,dt$$

with $x = \Phi^{-1}(s)$ and $y = \Phi^{-1}(t)$, ρ being the parameter of the Gaussian copula.

(2)

The t copula is defined as

$$T_{\rho,\vartheta}(u,v) = t_{\rho,\vartheta}(t^{-1}(u), t^{-1}(v))$$

$$= \int_{-\infty}^{t^{-1}(u)} \int_{-\infty}^{t^{-1}(v)} \frac{1}{2\pi(\sqrt{1-\rho^2})} \left(1 + \frac{s^2 + t^2 - 2\rho st}{\vartheta(1-\rho^2)}\right)^{-(\vartheta+2)/2} ds\,dt$$

(3)

The parameters of the copula are the correlation coefficient ρ and the degree of freedom ϑ.

An Archimedean copula is given by

$$C^A(u,v) = \varphi^{[-1]}(\varphi^{-1}(u) + \varphi^{-1}(v))$$

(4)

The function φ is called a generator of the copula with dependence parameter α. The table below gives generator function, the copula density and the range of the parameter α for the well known copulas.

Copula	Generator function $\varphi_\alpha(t)$	C(u,v)	Range for α
Gumbel	$(-\ln t)^\alpha$	$\exp\left\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{\frac{1}{\alpha}}\right\}$	$[1, +\infty)$
Clayton	$\frac{1}{\alpha}(t^{-\alpha} - 1)$	$\text{Max}[(u^{-\alpha} + v^{-\alpha} -1)^{-1/\alpha}, 0]$	$[-1,0) \cup (0,+\infty]$
Frank	$-\ln\frac{e^{-\alpha}-1}{e^{-\alpha}-1}$	$-\frac{1}{\alpha}\ln\left[1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1}\right]$	$(-\infty,0) \cup (0, +\infty)$

3. Marginal Distributions for MUR/USD and MUR/INR Exchange Rates

In this section, we identify the best fitting probability distribution for each of the weekly exchange rate series MUR/USD and MUR/INR. Geometric returns for both the series from 7th Nov 1993 to 28th Feb 2011 are obtained so as to make the series consistent and remove seasonality. Let

$$rx_1 = \ln\left[\frac{(MUR/USD)_t}{(MUR/USD)_{t-1}}\right] \text{ and } rx_2 = \ln\left[\frac{(MUR/INR)_t}{(MUR/INR)_{t-1}}\right]$$ be the weekly geometric returns. The

summary statistics computed from 903 observations from each series are provided in Table 1.

**Table 1. Summary statistics of rx_1 and rx_2**

Statistic	rx_1	rx_2
Standard deviation	0.0071	0.0086
Skewness	-0.6160	-0.6234
Kurtosis	5.6008	8.5582
ADF critical value	-9.0022	-9.6248
p-value for ADF test	0.01	0.01

Summary statistics suggest that both series are negatively skewed and leptokurtic with very low variance. Augmented Dickey Fuller test result shows that both series are stationary. It is obvious that marginal distributions can't be Gaussian. Six well-known distributions are fitted to both the series and on the basis of Akaike Information Criterion (AIC) values provided in Table 2, it is concluded that hyperbolic distribution provides the best fit to both series and normal is the worst. Figure 1 also confirms the good fit of hyperbolic distribution to these series. Hyperbolic distribution is in fact able to account for the heavy tails as well as the skewness in the data.

Table 2. Fitting of distributions to geometric returns of exchange rates

	Fit to rx_1	Fit to rx_2
Distribution	-AIC	-AIC
Hyperbolic	6599.150	6284.186
Cauchy	6459.221	6196.627
Normal	6350.085	6013.985
Beta 4	6357.282	6018.842
Extreme Value Maximum	5767.962	5280.636
Student t	6552.900	6275.960

Figure 1. Hyperbolic fit to marginal series
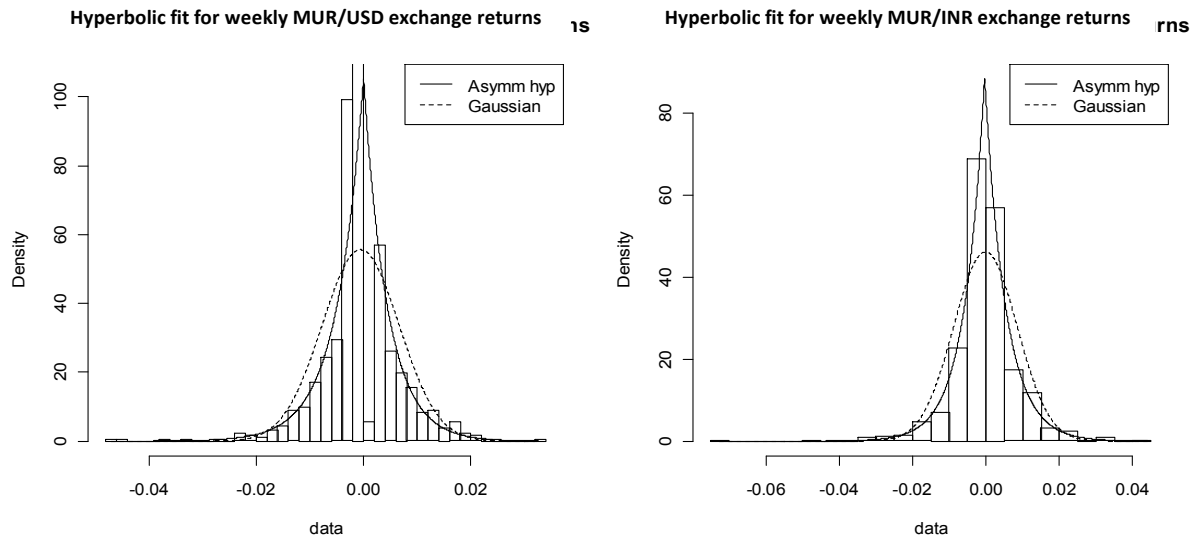
4. Fitting the Copula Models to Bivariate Exchange Rates

In this section, we fit the Elliptical and Archimedean copulas discussed in section 2, to model the dependence structure between MUR/USD and MUR/INR. R software is used for fitting the copulas. Specifically, from the fitted hyperbolic marginals, we first generate pseudo samples in the unit interval of [0,1]. The scatter plots of the real data and the pseudo data are shown in Figure 2. Using an approach similar to Dias et Embrechts (2009), we then simulate each copula for the bivariate exchange rate data and obtain maximum likelihood estimates of the corresponding copula parameters and information criterion values (AIC, SIC and HQIC).

Figure 2. Scatter Plots

Plot of actual returns

Plot of Pseudo Samples

These values together with the values of dependence parameters are presented in Table 3. On the basis of information criteria, t copula gives the best fit followed by Gumbel, Frank, Gaussian and Clayton. t copula as shown in Figure 3, is capable of modeling the tail dependence.

Table 3. Copula Fit Results

Copula	Estimated parameters	-AIC	-SIC	-HQIC	Spearman's Rho	Tail index	
						Lower	Upper
Gaussian	0.702	605	601	604	0.675	0	0
t	0.756, 2.234	820	810	816	0.740	0.547	0.597
Clayton	1.347	458	448	454	0.567	0.598	0
Frank	6.962	654	644	650	0.761	0	0
Gumbel	2.044	671	662	668	0.695	0	0.596

Figure 3. t copula fit

t copula with rho=0.7561 and nu = 2.2343
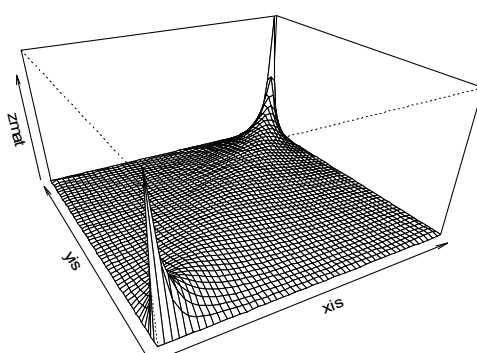
5. Conclusion

The dependence between the exchange rates of USD and INR with respect to MUR is modeled using copula approach. The two exchange rate series are marginally asymmetric as well as leptokurtic. The information criteria justify the fit of hyperbolic distribution to each series. Elliptical copulas and the Archimedean copulas: Frank, Clayton and Gumbel are then fitted to the pseudo bivariate series generated from hyperbolic distribution. On the basis of three information criteria based on log likelihoods, it is concluded that t copula is the best choice to explain the dependence between the MUR/USD and MUR/INR series.

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