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Capital Asset Pricing Model with Frictions: Application to the Tunisian Stock Market

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ABSTRACT: For many decades, asset-pricing theory was constructed on absence of frictions hypothesis. However, since pioneers' works of Chen, Kim and Kon (1975) and Amihud and Mendelson (1986) in microstructure area, systematic violations of this assumption were observed and the standard CAPM generates biased estimates of returns. The main obstacle that we face when we use CAPM with frictions is the unavailability of the observable variables that directly measure illiquidity costs, transaction costs and information costs. Microstructure literature provides many models to estimate them. We use Amihud (2002) model to estimate illiquidity costs and Lin et al. (1995) model to estimate transaction and information costs. Before a study of a sample of 40 quoted securities in Tunisian financial market, on the period of 07/02/2011 to 31/01/2013, results appear conclusive. First, we show the existence of asset pricing bias compared to the standard CAPM. Furthermore, we find a strong relation between transaction costs, information costs and illiquidity estimators and returns; thus the use of the CAPM with frictions. This model appears very strong and the results prove the necessity to account for frictions in asset pricing.

Keywords: CAPM; market microstructure; frictions; bid-ask spread components.
JEL Classifications: G20; G21; G01; G31; G32

1. Introduction

The Efficient Market Hypothesis was formulated in the United States at the beginning of the sixties within the famous "School of Chicago". The first empirical studies intended to validate it were made to «Center for Research in Security Prices ". The results of the first tests of efficiency realized on the North American market during the sixties and first years of the next decade showed themselves massively favorable to the hypothesis of the efficiency. The results obtained afterward have proved much more contrasted.

Besides, markets microstructure theory is interested to study the impact of the various markets frictions and the tendencies of the asymmetric information about the price formation process. The central idea of the microstructure theory is that the assets prices cannot be planned by manipulating complete information because of the variety of the frictions on markets (cool, tax system, transaction, information and illiquidity costs…). These frictions drive to a price formation which becomes, since the works of Demsetz (1968), the central theme of the microstructure theory.

The bid-ask spread is the difference between the seller price (ask) and the buyer price (bid). Several researchers (such as: Glosten and Harris (1988); Hasbrouck (1991) and Huang and Stoll (1997) decomposes the spread into an informative part and another one not informative. In the first part, called the adverse information cost, the bid-ask spread constitutes a compensation of the potential losses which market maker can undergo in front of better informed operators. In the second part (not informative) we can distinguish the orders processing costs (recording services and control) and the inventory holding costs (the compensation costs so that market maker accept to hold no preferment portfolio).

Traditionally, the financial assets pricing literature is essentially established on economic factors which influence the assets prices. Recently, the microstructure literature suggests the existence of other factors which are related with the characteristics of the markets that assets are negotiated. The microstructure theory proves the existence of the imperfections or frictions markets. The major foundation of this extensive literature is the process by which the information is incorporated into the prices. The microstructure literature supplies structural models about price and volatility efficiency.

The effects of taxes and transaction costs on asset pricing are presented, first, by Black (1974) who shows that taxes discourage certain investors to invest in certain assets and in certain countries. Stulz (1981) proposes a model for which the detention of the foreign assets is very expensive. He shows that because of higher costs, certain securities are not the object of exchanges and the foreign investors tend to hold more domestic securities. This is implies the existence of evaluation bias in the traditional CAPM.

The idea that returns depend on characteristics of the exchange process was differently examined in the literature. The most important study is the one of Amihud and Mendelson (1986) that assert that the liquidity, measured by the bid-ask spread, affects the securities returns. They explain this by the fact that in equilibrium, the traders are going to require higher returns to hold securities with wide bid-ask spread.

The purpose looked for this paper is to examine the relative problem in search of a modeling, based on the microstructure theory of the, getting closer to the realities of the stock markets who allows to improve the apprehension of the fundamental elements governing the dynamics of the prices of financial assets (price formation).

To answer empirically our problem, we adopted the following methodology:First, we tested the validity of standard CAPM. Secondly we estimated the illiquidity, transaction and information costs, while examining their effects on the securities returns. Thirdly, we tested empirically, on Tunisian stock market, the validity of two models based on microstructure theory: the CAPM with transaction and information costs and the CAPM with illiquidity costs. Finally, we tried to identify a CAPM with frictions suited to the Tunisian stock market.

This paper articulates around three sections. Section 2, presents a literature review developing some pioneers models of valuation of financial assets (CAPM-SLM) taking into account frictions of the market. Section 3, proposes a theoretical extension of the standard CAPM by presenting two models based on the markets microstructure, namely: the CAPM with transactions and information costs (see Aboura and Bellalah, 2001) and the CAPM with illiquidity costs (Liquidity-adjusted asset pricing model; LACAPM proposed by Viral et al., 2004). The section 4, supplies the database and the estimations results of econometrics models used in our empirical study.

2. Pioneers Models on Asset Pricing Taking into Account Frictions Markets: Review Literature

The works which are interested to study the effect of markets frictions on the asset pricing and portfolio choice are three types: purely theoretical works, works which are at the same time theoretical and empirical, and purely empirical works.

The first works type is made by Chen et al. (1975), Magill and Constantinides (1976), Constantinides (1986), Amihud et al. (1992) and Heidle (1999). Their works were based on the portfolio choice theory at continuous time, by taking into account transaction costs. Chen et al. (1975) and Amihud et al. (1992) show that transaction costs have significant effects on the asset pricing. Whereas, Constantinides (1986) concludes that transaction costs have a second-class effect on the premium of liquidity implied by equilibrium assets returns. This means that the effect of small firms cannot explain the effect of transaction costs, what is contradictory to the empirical results of Stoll & Whaley (1983) and Amihud and Mendelson (1986). Heidle (1999) comments the empirical study of AM-86 and mentions that the latter tested only their second proposal, since their first proposal (1) on the clientele effect is difficult to make out to test. The clientele effect can be explained by the fact that the assets which possess the highest bid-ask spread tend to be held by investors who have long periods of detention. Assets held by these investors are going to have high returns and high transaction costs.

However, test this first proposal demand the obtaining of data over the detention periods of assets by the investors.

The second type of works is made by Amihud and Mendelson (1986), Brennan and Subrahmanyam (1995) and James and Woodward (1995). The most important study is the one of Amihud and Mendelson (1986) who examine the effect of the illiquidity, measured by the bid-ask spread, on the assets returns. In their model, the authors worked on periods different from detention of assets and for investors who hold assets with different ask-bid spreads. They prove that the expected returns increase with enlargement bid-ask spread and that there is a clientele effect. This clientele effect can be explained by the fact that the investors with long detention periods of assets will rather hold assets which undergo high transaction costs and, consequently, they are going to require high returns.

The third type of works is developed by Amihud and Mendelson (1986, 1989), Stoll and Whaley (1983), Day et al. (1985), Reinganum (1990), Eleswarapu and Reinganum (1993) and Falkenstein (1996). With the exception of the study of Eleswarapu and Reinganum (1993), all the empirical studies support the idea according to which the transaction cost is the main determiner of the assets returns and which the size effect (of small firms) can be explained by the transaction costs. Eleswarapu and Reinganum (1993) discover a certain ambiguity on this point of view by analyzing the seasonality. Amihud and Mendelson (1986) tests empirically the hypothesis according to which the assets returns are in positive and concave relation with bid ask spread. The study proposed by AM-86 indeed confirms this hypothesis. Falkenstein (1996) shows that the investors prefer to hold assets which undergo moderate transaction costs and volatility. He also shows that the investors tend to exchange according to the availability of the information. In their model, the information is supposed to collect by the investors via the publication of the age of assets and publication of new information.

3. Theoretical Extension of the Standard CAPM

The standard CAPM developed by Sharp (1964), Lintner (1965) and Mossin (1966) is without a doubt the cornerstone of modern finance. The development of the standard CAPM assumes that all investors have the same expectations about the distributions of securities returns. However, in practice, investors do not necessarily have the same information. In fact, it is even widely recognized that these are the differences of investor's opinionsthat generate most of the transactions on financial markets. There are many researchers who have contributed to release the hypothesis of the absence of transaction costs. Indeed, of the mid-1970s a large number of extensions of CAPM were derived.
Microstructure theory comes to support these extensions, by proposing pricing models with frictions, such as: illiquidity, transactions and information costs.

3.1. CAPM with transaction and information costs proposed by Aboura and Bellalah (2001)

In equilibrium, the assets returns are given by the following equation:

$$\tilde{\tilde{R}}_k = \bar{R}_k + b_k \tilde{\tilde{Y}} + \sigma_k \tilde{\varepsilon} \qquad \textbf{(A)}$$

$\tilde{\tilde{R}}_k$: The expected rate of return of security k

$\tilde{\tilde{Y}}$: A random variable common factor with: $E(\tilde{Y})=0$ and $E(Y^2)=1$

$$E(\tilde{\varepsilon}_k / \varepsilon_1 \ldots\ldots\ldots \varepsilon_{k-1}, \varepsilon_{k+1} \ldots\ldots \varepsilon_n, Y) = 0 \quad \textbf{for k=1,2........n}$$

Equation (A) is similar to the diagonal model from Sharp (1964) and the version on a single factor of APT (Asset Pricing Theory) proposed by Ross (1976) and Merton (1987).
In addition to the n risky securities, they assume the existence of risk-free assets. The rate of return on this asset is given by the equation:

$$\tilde{\tilde{R}}_{n+1} = R$$

The model assumes the existence of transaction costs or taxes on the exchange of assets (see Black [1974] and Lewis [1999]) and that the purchases and short sales are unlimited. Investors are showers at risk and portfolio selection occurs in the way of Markowitz-Tobin (1959) that use the mean-Variance criterion.The preference of the investor j is represented by the following equation:

$$U^{j} = E(\tilde{\tilde{R}}^{j} W^{j}) - \frac{\delta^{j}}{2W^{j}} Var(\tilde{\tilde{R}}^{j} W^{j})$$

W^{j} **:** The wealth of investor j;

$\tilde{\tilde{R}}^{j}$: The return on his portfolio of investor j;

$\delta^{j} \geq 0$, for j = 1, 2……..N.

Let ω_k^{j} be the fraction of initial wealth allocated to security k by investor j. The return on portfolio for an investor j in the presence of transaction costs can be written as follows:

$$\tilde{\tilde{R}}^{j} = \sum_{k=1}^{n} \omega_k^{j} (\tilde{\tilde{R}}_k - r_k) + \omega_{n+1}^{j} R$$

r_k , is the transaction costs paid by investor j on asset k.

After several iterations of this model and inserting the transaction and information costs, Bellalah &Aboura (2001) lead to the final model that gives the expected rate of the security return as a function of the risk free rate, the transaction costs, the shadow cost of incomplete information and the risk premium. The model is given by the following equation:

$$\bar{R}_k = R + r_k + \lambda_k + \beta_k(\tilde{\tilde{R}}_M - R - \lambda_M - r_M)$$

r_k : The transaction costs paid by investor j on asset k.

R : The risk-free interest rate.

β_k : The beta of asset k.

r_M : The aggregate market transaction costs.

λ_k : The information costs paid by investor j on asset k.

λ_M : The aggregate market information costs.

3.2. Presentation Model Liquidity-Adjusted Capital Asset Pricing Model (LACAPM)

This model, proposed by Achraya and Pedersen (2004), we offer a theoretical framework that illustrates several channels through which the risk and cost of illiquidity may affect the formation of asset prices. The model is based on the following assumptions:

• The model assumes the existence of a simple economy of several generations of agents bounded over periods t ∈]…; -2 ; -1 ; 0 ; 1 ; 2 ; …[. The generation of N, t consists agents, indexed by n, living for two periods t and t +1.

• The agent n generating your wealth at time t and has no other source of income. It makes exchanges in periods t and t +1 and has a utility function that depends on T 1 consumption. It has a constant absolute risk aversion, A_n, so that preferences are represented by the following expected utility function:

$$-E_t \exp(-A^{n} x_{t+1})$$

Where x_{t+1} is consumption at time t +1.

• There I titles indexed by i = 1, ..., t, ..., I, with a total S^ishares of stock i. At time t, the title i pays a dividend, and has said during ex-dividend pit and illiquidity cost of C^i_t; whereD^i_t and $C^i_t$are random variables. All variables on a probability $\{\Omega, F, P\}$ space; and all random variables indexed by t are measurable filtration $\{F_t\}$ representing the information generally available to investors.

The cost of illiquidity, C^i_t; is modeled simply by the cost per title i sold. Therefore, agents can buy in p_i but must sell p_i -C_i. Short sales are not allowed.

• In this model, uncertainty in the cost of illiquidity which generates liquidity risk. Specifically, the model assumes that $D^i_t$and $C^i_t$are autoregressive process of order 1:

$$D_t = \bar{D} + \rho^{D}(D_{t-1} - \bar{D}) + \varepsilon_t$$

$$C_t = \bar{C} + \rho^{D}(C_{t-1} - \bar{C}) + \eta_t$$

$\bar{D}, \bar{C} \in \Re^{1}_{+}$: Vectors are positive.

$\rho^{C}, \rho^{D} \in [0,1]$.

ε_t, η_t : Are independent and identically distributed normal process.

- The model assumes that agents can borrow and lend at the risk-free rate rf> 1.

Assumptions about agents, preferences, and dividends are strong. The central hypothesis is the fact that the usefulness of the quadratic function is increasing and concave. These assumptions are also used to study the predictability caused by illiquidity and co-variations in yields and liquidity performance.

It should also be noted that the estimated cost of illiquidity, C_{it}, the transaction cost or the bid-ask spread is not realistic. Indeed, the cost of illiquidity could represent other actual expenses; for example, those related to the execution of swap transactions (see Duffie, Grleanu, and Pedersen (2003)). The gross assets expected return of a can be formulated as follows:

$$r_t^i = \frac{D_t^i + P_i^t}{P_{t-1}^i}$$

The relative illiquidity cost can be calculated as follows:

$$c_t^i = \frac{C_i^t}{P_{t-1}^i}$$

Market performance can be formulated as follows:

$$r_t^M = \frac{\sum_i S^i (D_t^i + P_i^t)}{\sum_i s^i P_{t-1}^i}$$

The cost of market illiquidity is measured by (proposed by Amihud(2002) measures): $c_t^M = \frac{\sum_i S^i C_i^t}{\sum_i S^i P_{t-1}^i}$

In a competitive equilibrium agent choose consumption and portfolios that maximize their utility functions and prices are determined by the market. To determine the equilibrium price, the authors assume the existence of an economy with the same agents in which the security pays a dividend, D_i- C_i, and costless liquidity. In this case, the results imply the validity of the standard CAPM (Markowitz, 1952, Sharpe, 1964, Lintner 1965, Mossin 1966). The authors suggest, however, that the balance in a real economy considering price frictions are identical to those suggested in our economy. This is consistent with the following two facts: (i) the net return to a long position is the same in both typessavings and (ii) all investors, in this case, take a long position in the market portfolio and a long or short position in the riskless asset. Therefore, the equilibrium return of the investor in the economy without frictions is feasible for the real economy and is also great.

These arguments show the passage of the standard CAPM, which assumes the absence of friction, to a CAPM with net returns of liquidity costs. By replacing the expression of standard CAPM in terms of net returns an expression in terms of gross returns, we get the CAPM adjusted liquidity. This is the main implication of this model.

At equilibrium, the net expected return conditional title i, is:

$$E_t(r_{t+1}^i - c_{t+1}^i) = r_f + \lambda_t \frac{\text{cov}(r_{t+1}^i - c_{t+1}^i, r_{t+1}^M - c_{t+1}^M)}{\text{var}(r_{t+1}^M - c_{t+1}^M)}$$

With , $\lambda_t = E_t(r_{t+1}^M - c_{t+1}^M - r_f)$: the liquidity risk premium.

The last equation shows that the excess of the required relative to the risk-free rate yield and the cost of liquidity depend on the covariance between the return on a security and liquidity of the market.The most interesting results emerged from the empirical examination of this model Achraya & Pedersen (2004) are:

- An illiquid stock has a high risk of liquidity. This result confirms the race for liquidity (" flight to liquidity ") observed in illiquid markets.

- Empirical tests have shown that the sensitivity of the liquidity of a security to the profitability of market risk is the source of the largest liquidity.

- The authors show that the CAPM risk-adjusted liquidity has a higher than standard CAPM (R2 = 91 %> R2 = 75.7%) for the same degree of freedom explanatory power. The model improvement is not due to the addition of an explanatory variable but better specification.

4. Empirical Evidence

The data used in this work come from website tustex.com.tn and a company of market intermediation. We have retained as sample 40 securities quoted in continuous on the Tunisian stock market. These securities are selected according to criteria of market capitalization and number of day's quotations. Data concerns daily closing prices, the best prices and offered and demandedquantities, the transactions volume and the number of transactions. It should note that we are going to exclude Saturdays, Sundays, day holidays and the days for which the securities were not quoted. The study is conducted on the period going February 07, 2011 until December 31, 2013.

This research task provides an empirical study applied to the models of assets pricing with frictions. With this intention, we proceeded as follows: initially, we tested the validity of standard CAPM model. Then, we estimated the illiquidity, transaction and information costs, while examining their effects on the assets returns. Lastly, we tested the CAPM with transaction and information costs and, then, the CAPM with illiquidity costs proposed byAchraya & Pedersen (2004).

4.1. Empirical Validation of standard CAPM

The standard CAPM is a model of evaluation in equilibrium which makes it possible to visualize the existing relation between the assets returns excess (compared to the rate without risk) and the market portfolio returns (or the systematic risk). The standard CAPM is presented as follows:

$$E(R_{i,t})-R_{f,t}=\beta_i\left[E(R_{m,t})-R_{f,t}\right]+\varepsilon_{i,t} \textbf{ où, } \varepsilon_{i,t}\rightarrow iid(0,\sigma^2) \quad (1)$$

$E(R_{i,t})$: Expected returns portfolio or title i at time t.

$E(R_{m,t})$: Expected returns from the portfolio market at the time t.

$R_{f,t}$: The rate of risk-free return for the period of investment

β_i : The sensitivity coefficient of portfolio or title i of the market portfolio.

For ends of estimate, it is convenient to transform the standard version of CAPM into an equivalent expression based on observable variables, namely:

$$R_{i,t}-R_{f,t}=\beta_i\left[R_{m,t}-R_{f,t}\right]+\varepsilon_{i,t} \quad (2)$$

Most empirical work carried in this field introduces a constant with this last expression:

$$R_{i,t}-R_{f,t}=\alpha_i+\beta_i\left[R_{m,t}-R_{f,t}\right]+\varepsilon_{i,t} \quad (3)$$

According to waitings of this equilibrium equation, the coefficients α must be statistically null.On the other hand, the coefficients β will be positive and significant. In short, to test the standard CAPM on the Tunisian stock market, we must estimate expression given by the equation (3). With this intention, we calculated monthly variables as follow:

- $R_{i,t}=(P_t-P_{t-1})/P_{t-1}$: Where, P_t, is the closing price relative to month t.
- $R_{m,t}=(I_t-I_{t-1})/I_{t-1}$, where I_t, is the closing price of BVMT score to month t
- $R_{f,t}=(1+R_{f,at})^{1/12}-1$ Where, $R_{f,at}$, is the annual balanced average rate of the subscription for the treasury bills transferable relating to the month t (all confused expiries).

The econometric technique used in this study is to estimate the regression given by equation (3) by OLS on panel data. The estimation results are presented in the table 1

Table1. Empirical Validation of the standard CAPM: $R_{i,t}-R_{f,t}=\alpha_i+\beta_i\left[R_{m,t}-R_{f,t}\right]+\varepsilon_{i,t}$

α	t-student	β	t-student	R^2	F-stat
-0.001739	**-6.770873**	**0.606189**	**9.794946**	**0.121031**	**3.389582**

While referring to the statistics of Student, we notice clearly that the coefficient β , is positive and significant with the level of 1%. This validates waitings of standard CAPM and implies that the systematic risk plays aimportant role in the investors remuneration. In the same way, the coefficient α_i is statistically not null with the level of 1%. This is in contradiction with waiting's of standard CAPM and implies that the constant intervenes significantly in the explanation securities returns excess quoted in continuous on the Tunisian stock market. Moreover, the determination coefficient R^2 is a little low (12%). This justified the existence of an anomaly.

The existence of such an anomaly requires us to think of introducing to the standard CAPM other factors which can significantly influence the excess returns of the securities stock exchange. These factors can be at the origin of several sources, such as: liquidity, illiquidity risk, asymmetric information cost.

4.2. Estimation of illiquidity, transaction and information costs and their impact on returns

4.2.1) Estimation of transaction and information costs,

Recently, several research studies have presented statistical models to estimate and decompose the bid-ask spread. These models can be divided into two broad classes. The first class of models is introduced by Roll (1984) which was one of the first to propose a simple estimator for the bid-ask spread. This estimator is established on the covariance of returns, since the real transactions are either at best price offered (ask), or at the best price demanded (bid). The models of Choi et al. (1988) and George et al. (1991) are in conformity to the model of Roll (1984). Another class of models is based on the idea that bid-ask spread depends on indicators of the trade. These indicators of trade models are driven by the arrival of orders and the response of prices to the arrival of these orders. These models include: Glosten and Harris (1988), Stoll (1989), Hasbrouck (1991), Easley et al. (1996) and Huang et al. (1997).

The underlying main argument to all the models of adverse selection is the enlargement of the bid-ask spread due to the presence of informed agents on the financial markets. On microstructure theory, the notion of spread is very important because it represents "the price of immediacy" as well as transaction and information costs in market.

So far, the discussion of the relevance of the adverse selection theory is limited to only markets led by price. The problem of the application of the models presented previously lies by the fact that there is no actual empirical model of decomposition bid-ask spread in a market driven by orders (order-driven Market). This suggests, therefore, that these models are not suited to this type of market simply because there is no supplier of liquidity in a broad sense.

In addition, some scientists (such as: Brockman and Chung (1999) and Majois and Winne (2003)) nevertheless tried themselves to transpose these models in the universe from the market directed by the orders.

Among the models of decomposition of bid-ask spread presented previously, our choice is related tothe decomposition suggested by Lin et al. (1995) mainly for three reasons. First,this model enables us to provide, at the same time, two useful estimators following ourstudy, namely: the transaction and information costs. Then, this model does not supposethe existence of aninventory costs component. Since our study relates to a marketdirected by the orders, this choice seemsadequate. Lastly, this model seems to be most robust concerning its estimate (See Clarck & Shastri; 2001).

For purposes of estimating we transformed the LSB 95 model in logarithmic form (see, also, Heflin andShaw, 2000):

$$\Delta \log \left[M_{i,t+1}\right] = \lambda_i Z_{i,t} + \varepsilon_{i,t} \qquad (4)$$

$$Z_{i,t+1} = \theta_i Z_{i,t} + \eta_{i,t} \qquad (5)$$

$$\gamma_i = 1 - \lambda_i - \theta_i \qquad (6)$$

$M_{i,t} = \dfrac{ask_{i,t} + bid_{i,t}}{2}$: The middle bid-ask spreadof a security i at time t.

$Z_{i,t} = \log(P_{i,t}) - \log(M_{i,t})$: The half effective spread of a security i at time t.

; the difference between transaction price, log (P_t) andthe middle bid-ask spread, log (M_t).

λ : Measure the component of adverse selection of a security i at time t, expressed as a percentage of the relative bid-ask spread.

θ: Measure the orders persistence of the security at time t, expressed as a percentage of the relative bid-ask spread.
γ : Measure the transaction costs of a security i at time t, expressed as a percentage of the relative bid-ask spread.

To calculate the monthly components of securities bid–ask spread, equations (4) and (5) were estimated by OLS on daily time series of three months (63 days). The study period extends from February 07, 2011 to January 31, 2013. In total, we have been carried out 320 (8 quarter * 40 securities) estimates for each equation to extract monthly shares of each component in bid-ask spread. The estimates results are summarized in the figure 1. The value of (m), (tr) and (z) retained for the 40 securities in our sample, correspond to the average of the monthly values throughout the period of our study.

Figure 1. Decomposition of bid-ask spread according to LSB 95 model

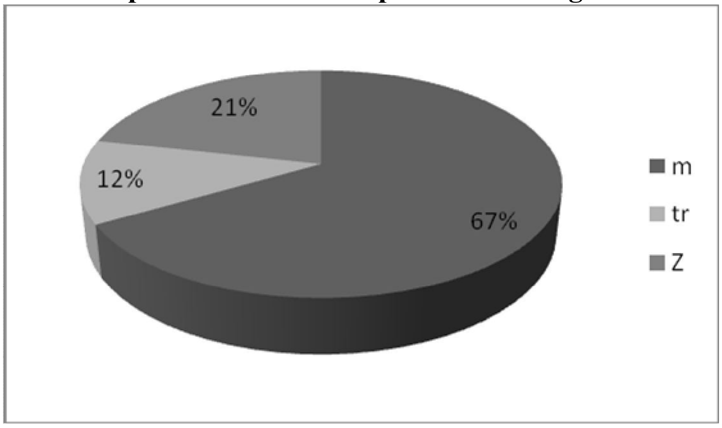

Figure 1 indicates that the cost generated by the adverse selection cost is estimated at 67%, the transaction cost at 12% and the orders persistence at 21% of the bid-ask spread. This reversed the idea that transaction and information costs constitute the totality of the spread.Finally, we generate, for each security in our sample, monthly series on the transaction and information costs by multiplying the coefficients (m, tr and z) by the relative monthly of bid-ask spread, noted by:

$$FAR_{i,t} = \frac{ask_{i,t} - bid_{i,t}}{\left[(ask_{i,t} + bid_{i,t})/2\right]}$$

4.2.2) Impact of transaction, information and illiquidity costs on the returns

Before empirically testing the various CAPM with frictions, we, initially, study the effect of various costs on the securities returns. With this intention, we will consider the following models:

$$R_{i,t} = a_{0,i} + a_{1,i} m_{i,t} + \upsilon_{i,t} \quad \textbf{(7)}$$

$$R_{i,t} = a_{0,i} + a_{1,i} tr_{i,t} + \upsilon_{i,t} \quad \textbf{(8)}$$

$$R_{i,t} = a_{0,i} + a_{1,i} illiq_{i,t} + \upsilon_{i,t} \quad \textbf{(9)}$$

$R_{it} = \ln(P_t / P_{t-1})$

m_{it} :Information costsof security i at time t.

tr_{it} :Transaction costs of security i at time t.

$illiq_{it} = \frac{1}{Days_{it}} \sum_{d=1}^{Days_{it}} \frac{|\ln(P_{dit} / P_{dit-1})|}{(V_{dit})}$: Illiquidity costs of security i at time t.

The estimates results of models (7), (8) and (9) on the panel data are presented in the table 2.

Table 2. Frictions effect on returns

PANEL	A0	t-stat	A1	t-stat	R^2
m	**-0.001022**	**-6.013087**	**0.069146**	**6.889126**	**0.047202**
tr	**-5.78E-05**	**-0.515076**	**-0.097474**	**-6.832283**	**0.046463**
illiq	-0.001227	-4.595090	0.595578	6.098638	0.078325

The results presented in table 2 display a coefficient, related to the information costs (m), positive and significant to the level of 1%. This shows, probably, that the information costs estimated by LSB 95 model can contribute to explain the securities returns.With regard to the effect of transaction cost on the securities returns, table 2 appears a negative and significant coefficient with the level of 1%. This suggests that transaction costs have a significant effect on returns.

Moreover, the results in table 2 go exactly in the same direction as the theoretical literature indicating a negative relation between the liquidity and the securities returns (see, for example, Pastor and Stambaugh (2003)). The sign expected from coefficient a_1 is positive because an increase in illiquidity cost implies compensation by an increase in the securities returns.

It should be noted that the determination coefficients is low for thethree models. This suggests the existence of other factors explaining the stock exchange returns.

4.3 Empirical Validation of CAPM with friction

4.3.1 Empirical Validation of CAPM with transaction costs and information

The objective of this sub-section is to empirically validate the theoretical model of asset pricing taking into account transaction costs and information.

$$\bar{R}_k = R + a_k m_k + b_k tr_k + \beta_k (\tilde{R}_M - R - m_M - tr_M) \qquad (10)$$

For estimation purposes, we instead estimated the following model:

$$R_{i,t} - R_{f,t} = \alpha_i + a_i m_{i,t} + b_i tr_{i,t} + \beta_i \lfloor R_{m,t} - R_{f,t} - m_{m,t} - tr_{m,t} \rfloor + \varepsilon_{i,t} \qquad (11)$$

According to the expectations of the balance equation, the coefficients should be statistically zero. In contrast, the coefficients will be significant and positive to reflect a positive relationship between excess returns (over the risk free rate, the cost of transaction and information costs) and net systematic risk of transaction costs and information. The estimation results, OLS, from equation (11), using data of the panel, the software Eviews 6.1 are presented in the table 3.

Table 3. Estimation of the CAPM with transaction costs and information

α	t-Statistic	A	t-Statistic	b	t-Statistic	β	t-Statistic	R^2
-0.00131	-1.94178	0.041794	2.804151	-0.04525	-2.07421	0.172012	4.220862	0.183138

The results presented in Table 3 are consistent, likely expectations of this model. Indeed, on the one hand, the coefficient is positive and significant at the 1% level. On the other hand, the coefficients related to transaction costs are significant. In addition, the coefficient of determination R 2 is improved compared to the standard CAPM previously estimated, rising from 9% to 18%.

4.3.2 Empirical Validation with the CAPM cost of liquidity (Liquidity-Adjusted Capital Asset Pricing Model, 2004).

The objective of this sub-section is to empirically validate the theoretical model of asset pricing taking into account the illiquidity costs proposed by Acharya a Pedersen (2004):

$$\bar{R}_{i,t} - R_{f,t} = c_i + \alpha_i \, ILLIQ_{i,t} + \beta_k (\tilde{R}_M - R_{f,t} - ILLIQ_M) \qquad (12)$$

For estimation purposes, we instead estimated the following model:

$$R_{i,t} - R_{f,t} = c_i + \alpha_i ILLIQ_{i,t} + \beta_i \lfloor R_{m,t} - R_{f,t} - ILLIQ_{m,t} \rfloor + \varepsilon_{i,t} \qquad (13)$$

According to the expectations of the balance equation, the following factors should be statistically zero. In contrast, the coefficients $\beta_i$are positive and significant to reflect a positive relationship between excess returns (relative to the risk-free rate) and net systematic risk of illiquidity costs. As regards the coefficients α_i, the literature shows an ambiguity as to the relationship between liquidity and yield. The estimation results, OLS, from equation (13), using data of the panel, the software Eviews 6 are shown in the table 4.

Table 4. CAPM with cost and liquidity risk (LACAPM)

$$R_{i,t} - R_{f,t} = c_i + \alpha_i ILLIQ_{i,t} + \beta_i \lfloor R_{m,t} - R_{f,t} - ILLIQ_{m,t} \rfloor + \varepsilon_{i,t}$$

c	t-Statistic	α	t-Statistic	β	t-Statistic	R^2
-0.000335	-0.663047	0.667046	6.976422	0.804737	10.17497	0.270479

The results presented in Table 4 likely prove the validity of the CAPM into account liquidity. Indeed, first, the above coefficient is statistically zero. Secondly, the coefficient $\alpha_i$is positive and significant. This corroborates the results of Pastor and Stambaugh (2003) prove the existence of a negative relationship between yield and liquidity. Thirdly, the coefficient β_i is positive and significant at 1%. This corroborates the literature portfolio management indicating any type of market risk (the risk of the net market illiquidity cost in our case) will be composed by an additional return. Finally, the coefficient of determination R 2 is improved compared to the standard CAPM previously estimated (R^2 increased from 12 % to 27 %).

Altogether, we can conclude that the model is statistically LACAPM strongest (R2 Revenue Is the value 27%) compared to the standard CAPM and the CAPM with transaction costs and information. This encourages us to test our CAPM with frictions, namely:

$$R_{i,t} - R_{f,t} = \alpha_i + a_i m_{it} + b_i tr_{it} + c_i ILLIQ_{it} + \beta_i \lfloor R_{m,t} - R_{f,t} - m_{mt} - tr_{mt} - ILLIQ_{m,t} \rfloor + \varepsilon_{i,t} \quad (14)$$

The estimation results, OLS, from equation (14), using data of the panel, the software Eviews 6 are shown in the table 5.

Table 5. Estimation of the CAPM with friction

α	t-Stat	a	t-Stat	b	t-Stat	c	t-Stat	β	t-Stat	R^2
-0.0028	-3.4722	0.034453	2.352267	-0.0344	-1.5930	0.495775	9.770650	0.14808	4.050506	0. 3231

The results presented in Table 5 are very powerful. Indeed, on the one hand, and all coefficients significant at 1% and conform to the expectations of the model. In addition, the coefficient of determination R 2 is improved relative to other previously estimated CAPM. Moreover, in order to prove that our CAPM with friction has a better specification compared to other models, we reconcile the models through the Fisher statistic and the coefficient of determination.

Table 6. Comparison between the different models

PANEL	R^2	F-stat
CAPM standard	0.128566	25.94098
CAPM with tr costs and info.	0.183138	28.89548
LACAPM	0.270479	50.00731
CAPMwith friction	0. 323181	83.4995

In summary, the various models studied may help to explain the performance of share prices on the Tunisian market with an explanatory superiority for the CAPM with friction with a coefficient of determination of approximately 32% and a statistical Fisher 83,5 . This result shows that our CAPM with frictions has a better specification compared to other models.

5. Conclusion

The objective of this paper is to determine a suitable friction CAPM with the Tunisian stock market. This model is based on the standard CAPM while releasing the assumption of the absence of market frictions. These frictions are mainly due to the presence of informed investors and the importance of transaction costs and liquidity risk borne by investors in their dealings on the stock market. For this, we empirically tested a CAPM with transaction costs and information costs and CAPM with liquidity.

The estimation of these models is the problem of the existence of unobservable variables, ie, the transaction costs of information and liquidity, which require recourse to estimation models. Regarding the estimation of transaction costs and information, market microstructure literature offers a multitude of methods, namely Glosten and Harris (1988), Stoll (1989), Hasbrouck (1991), Lin et al. (1995), Easley et al. (1996) and Huang and Stoll (1997). Our choice is focused on the model of Lin et al. (1995) since this model is best suited to the realities of the Tunisian stock market order-driven (no market maker) . Moreover, in order to estimate the costs of illiquidity we used the model of Amihud (2002).

The results of the study of the impact of information costs on returns show the need to take account of friction in the evaluation of stocks. To test this prediction, we have, first, to prove the existence of an evaluation bias in standard CAPM. This evaluation bias can be explained by the importance of frictions on the Tunisian stock market. Second, we empirically tested a CAPM with transaction costs and information. The estimation results show that the excess returns of the market portfolio has a positive and significant effect on excess returns. In addition, the explanatory power of the model is improved compared to the standard CAPM. Thirdly, we tested empirically LACAPM model proposed by Acharya and Pedersen (2004), incorporating the costs of illiquidity. Indeed, the results of our study indicate that LACAPM model has a higher explanatory power than the standard CAPM and the CAPM with transaction costs and information.

These results lead us to derive our final model; CAPM with friction. The latter is a CAPM same time incorporating the costs of liquidity and transaction information. This model has a higher compared to other models with a coefficient of determination 32% explanatory power.

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