International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2019, 9(4), 18-24.

Does Chaos Matter in Financial Time Series Analysis?

Marisa Faggini[1]*, Bruna Bruno[1], Anna Parziale[2]

[1]Department of Economics and Statistics - DISES, University of Salerno, Italy, [2]Department of Law Sciences University of Salerno, Italy. *Email: mfaggini@unisa.it

Received: 18 March 2019 **Accepted:** 05 June 2019 **DOI:** https://doi.org/10.32479/ijefi.8058

ABSTRACT

The apparent randomness of financial market led some economists to approach chaos theory as a theoretical framework able to explain those fluctuations. This interest is because some nonlinear deterministic systems with few degrees of freedom create signals that mimic stochastic signals from the point of view of traditional time series analysis but with a deepener analysis performed by adequate tools could be chaotic. The aim of this paper is explorative in its nature, pointing to investigate chaos literature in order to grasp the difficulties typical of these applied researches and to see if something new is happening.

Keywords: Chaos Theory, Time Series, Financial Markets
JEL Classifications: C1, G1, F65

1. INTRODUCTION

For a long time, the efficient market hypothesis (EMH) has been the dominant theory used to explain the mechanism of stocks' prices formation. In this framework, the assumption of perfect rationality of investors with an unbiased behaviour, able to manage full and instantaneously any new information implies that the prices level is determined by all the information available to economic agents and any change is a consequence of new information entering in the market. Since the arrival of new information is random, as consequence any price change will be random (Pasca, 2015). Therefore, the logic of the random walk is that if the flow of information is immediately reflected in stock prices, then tomorrow's price change will reflect only tomorrow's news and will be independent of the price changes today. Consequently, stock returns are considered Independent and Identically Distributed (IID) random variables that move as a random walk (Aparicio et al., 1999; Gilmore, 1993; Kyrtsou et al., 2004; Muckley, 2004; Sewell et al., 1996; Varson and Doran 1995). Nevertheless, the restricted assumptions and the presence of irregularities in financial time series have led to criticise the validity of EMH theory. The main argument against was that a martingale model cannot explain anomalies of financial market such as high volatility, abnormal returns or the bubbles formation. Therefore, the statistical analysis of financial time series began to dismiss this model as a good solution for describing the price formation mechanism. In particular, many empirical studies showed that stochastic processes do not always describe the dynamics of financial series, so that price changes are not always random (Pasca, 2015). From this critique many other approaches have been developed.

Starting from Fama (1965) many authors confirmed empirically the existence of fat tails in the distribution of financial return series. Since then a multitude of models explaining the dynamics of financial returns emerged, primarily based on the assumption that the distribution of price returns follows a power law.

Some others, instead, showed that prices changes can be modelled by α-stable Levi processes or Pareto stable processes (Mandelbrot, 1963), often associated with fractals and fractal Brownian motion. Mandelbrot (1967) showed that irregular financial time series are scale invariant, mimicking a fractal behaviour. From this pioneering work, Peter (1994) proposed the fractal markets hypothesis as alternative to the EMH, highlighting that the liquidity[1] not the efficiency is the main driving force of market equilibrium.

The fractal market hypothesis (FMH) is based on the laws of non-linearity, chaos and complexity theories. If financial market is in the calm periods, both the fractal market hypothesis and efficient market hypothesis produce similar results. The difference among them are evident when the market becomes more turbulent. In this case, as the equilibrium upon which the Efficient Market Hypothesis is grounded, is vanishing, the fluctuations and movements of the market become non-continuous and this turbulence it could be quoted as chaotic. Therefore, if the efficient market hypothesis regards those behaviours as outliers, the fractal market hypothesis is able to describe them and, more in general, to take into account a vast variety of irregular and unexpected behaviours. In such a scenario, the utility of deterministic models in to determine the dynamics of financial price series comes under question, as the dynamics of financial time series resembles a stochastic process characterized by the seeming unpredictability of future trends.

This apparent randomness led some economists to approach the chaos theory as a theoretical framework able to explain financial market fluctuations. Much of this interest was inspired by the fact that some nonlinear deterministic systems with few degrees of freedom create signals that mimic stochastic signals from the point of view of traditional time series analysis, but with a deepener analysis performed by adequate tools could be chaotic (Klioutchnikova et al., 2017). In this view those sequences apparently random can be not random because they may rise from deterministic nonlinear dynamical systems and, thus chaotic.

Traditional economics literature tends to consider each market as converging towards a stable equilibrium, constantly perturbed by external shocks that trigger its dynamic behaviour and fluctuations. In chaotic models, nonlinear dynamics are internally generated and are not dependent upon exogenous shocks. Markets are significantly influenced by human behaviour and, therefore, they are considered complex and their dynamic, characterized by the large number of economic, financial, political and psychological variables that may fluctuate over time. It follows that stochastic models are unable to manage the massive amount of information needed for an accurate forecasting and an effective representation of the relationship existing between variables. In many cases, the selection of key variables is based just on the correlation with each other, on the relevance and degree of variance as well as on the neglecting of the time-relation of variables' cause and effect behaviour. This is a critical issue for the assessment of dynamic systems, especially when the analysis is also, focused on human behaviours, where learning and cognition capacities evolve through time.

1 This aspect is very important considering that the most dramatic crashes are caused by the lack of liquidity that constrains investors to accept any price whether it is fair or not.

After a brief review of chaos literature and its tools, this paper aims at pointing out the critical issues that affect the research on the topic, underlining that unfortunately, since when the first studies on financial time series were conducted, nothing new is happened and no new tools have been developed.

2. FINANCIAL DATA ANALYSIS: AN OVERVIEW

The approaches used to address the analysis of time series can be classified into two main categories: Linear and nonlinear. Moreover, to explain randomness observed in real-life data, economists have included stochastic considerations in their speculations.

Linear stochastic models, in particular the class of ARMA models, have been considered a practical tool for financial analysis and forecasting but they suffer from a number of serious shortcomings for studying financial fluctuations (Potter, 1995). They let just to generate realizations with symmetrical cyclical fluctuations, being not able to accommodate large shocks, shifting trends, and structural changes. Moreover, exogenous disturbances were superimposed upon usual linear deterministic models to mimic the financial time series, leaving, often, significant features unexamined and unexploited. Therefore, alternative answers have been searched in the nonlinear approach. The ARCH processes proposed by Engle (1982) and generalised by Bollerslev (1986) are nonlinear stochastic models that let to grasp the dynamics occurring within data and which might, otherwise, be obscured by systematic noise, time varying volatility, and non-stationary trends. It follows that these models are currently, used for analysing financial time series. Among the "ARCH-type" models Exponential GARCH, Asymmetric Power ARCH, Threshold GARCH, IGARCH, and FIGARCH are the most popular. They are grounded on the assumption that data are nonlinear stochastic functions of their past values. By using these models, researches on financial data pointed out a widespread stochastic non-linearity, even though the main effects seem to rise from the variances of the respective distributions. Nevertheless, some studies (Brock et al., 1991; Frank and Stengos, 1988) indicated that generalized ARCH models still show some evidence of nonlinearities in the data. What this nonlinearity is and how it should be modelled is still an open question. Chaos theory could allow for detecting this nonlinearity but using nonlinear deterministic models.

In the literature, there is no standard definition of chaos (Ditto and Munakata, 1995). However, it is possible to define it, outlining its typical features: Nonlinearity, dependence on initial conditions, and presence of a strange attractor. Based on these signs and in order to investigate the chaotic dynamics in the time series, tools as correlation dimension, Lyapunov exponent, and BDS test have been developed.

The correlation dimension test, developed in physics by Grassberger and Procaccia, (1983), is used for measuring the dimension of strange attractor. A pure stochastic process will spread all space as evolving, but an attractor will restrict the movements of a chaotic system. Moreover, in the long-run trajectories, which tend to converge on the attractor showing that global stability are sign of chaotic motion. A necessary but not sufficient condition in order to define a system as chaotic is that the attractor has a fractal dimension. The notion of dimension refers to the degree of complexity of a system expressed by the minimum number of variables needed to replicate the system (Schwartz and Yousefi, 2003). While the topological dimension is always an integer, a chaotic system has non-integer dimensionality called fractal dimension. The major advantage of correlation dimension is the simplicity of calculating but it provides necessary, but not sufficient conditions for testing the presence of chaos. Moreover, designed for very large and clean data sets, its application to short time series remains a tricky problem. Thus, data sets with few hundred or even few thousand observations might be inadequate and not fitting with this procedure (Ruelle, 1991).

The Lyapunov exponent provides a more useful characterisation of chaotic systems because unlike the correlation dimension, which estimates the complexity of a nonlinear system, it indicates a system's level of chaos. In particular, it measures average exponential divergence or convergence between trajectories that differ only in having an "infinitesimally small" difference in their initial conditions. Based on time evolution of these values, it can be positive or negative, but at least one exponent must be positive for classifying a system as chaotic. As in the case of correlation dimension, also the estimation of Lyapunov exponent requires a large number of observations.

The BDS[2] test by Brock et al. (1996) is not properly a test for chaos[3] but using the correlation dimension checks the much more restrictive null hypothesis that the series is independent and identically distributed. This test can detect the presence of some types of non-IID behaviours resulting from a non-stationarity of the series: A linear stochastic system (such as ARMA processes), a nonlinear stochastic system (such as ARCH/GARCH processes), or a nonlinear deterministic system, which could feature low-order chaos. Nevertheless, some of the most common problems of application of this test are related to the presence of the noise in financial data and the requirement of large data sets for obtaining a reliable analysis (Brock and Sayers, 1988).

In this scenario, the researches on chaos in finance have followed two different directions. The first one based on non-linear deterministic theoretical model and pointed to demonstrate that specific configurations can create chaotic behaviours; the second one pointed to test time series through specific tools designed for detecting chaotic behaviours.

From this latter point of view, in the 1989 Scheinkman and LeBaron published the first article on the application of chaos tests to financial data. Analysing the United States weekly returns on the Centre for Research in Security Prices (CRSP) with BDS test, the authors found out a strong evidence of nonlinearity and some evidence of chaos. From this paper, there was a flourishing of applications (Faggini and Parziale, 2016) aimed at detecting chaos in financial data. In the following table, the results of some financial applications are reported.

Table 1 depicts not only the tests mainly and widely used for detecting chaos in financial time series, as correlation dimension, Lyapunov exponent, and BDS test but also topological tools. In fact, recently, another kind of tests have been considered, the so-called topological tools (Faggini, 2014). These tests are typically intended to study the organisation of the strange attractor, and they include close returns plot and recurrence plot. They exploit an essential property of a chaotic system, e.g. the tendency of the time series to nearly, but not exactly, repeat themselves over time. This property is known as recurrence property. It has been successfully applied in order to detect chaos in experimental data, but it can also provide information about the underlying system, which is responsible for chaotic behaviour (Mindlin et al., 1990; Mindlin and Gilmore, 1992). This method is particularly fitting for the analysis of quite small data sets, being robust against the noise.

3. OPEN QUESTIONS

Chaos theory has attracted researchers for its ability to explain complicated behaviour by equations with only a few degrees of freedom, without assuming random forces acting on the system. The attractiveness of this new paradigm and the failure of standard time series methods have raised high expectations. Nevertheless, the problems related to the quality and the lack of acceptable amounts of data, the appropriate level of disaggregation, and the proper definition of methods used for detecting chaos created highly constrains to development of financial analysis based on this theory. In particular, the difficult to use chaos theory in financial market is a direct consequence of some problems related to the application of its tools to financial time series (Faggini and Parziale, 2016).

The algorithms summarised in the previous section have been developed to find out chaos in experimental data. Because physicists can often generate huge samples of high-quality data form laboratory experiments, they consider these algorithms as directly applicable to their research.

In economic time series, small and noisy data sets are more common. Therefore, correlation dimension, Lyapunov exponent, and BDS test designed for very large, clean data sets, was found to be problematical when applied to these time series. Data sets with only a few hundred or even a few thousand observations may be inadequate for these procedures[4] because the shortness and the noise may render any dimension calculation useless[5]. Therefore, testing financial series is often approached in a suspicious way because the gathered data are insufficient to get long sampling intervals (Hsieh, 1991), and involve mixed effects. In fact, not only the distinction between noise and nonlinearities must be

2 "Details of which may be found in Dechert (1996). Subsequent to its introduction, the BBS test has been generalised by Savit and Green (1991) and Wu et al. (1993) and more recently, DeLima (1998) introduced an iterative version of the BBS test" McKenzie (2001).

3 LeBaron (1994).

4 Ruelle (1991).

5 Brock and Sayers (1988).

Table 1: Results of researches of chaos in financial data

Year	Author(s)	Tools	Results
1989	Scheinkman and LeBaron	BDS test	Evidence of chaos
1991	Blank	Correlation dimensions and Lyapunov tests	Evidence of chaos
1991	Hsieh	BDS test	Evidence of chaos
1992	DeCoster et al.	Correlation dimensions	Evidence of chaos
1992	Vaidyanathan and Krehbiel	Correlation dimensions and Lyapunov test	Evidence of chaos
1992	Mayfield and Mizrach	Correlation dimensions and Lyapunov test	Evidence of chaos
1994	Brorsen and Yang	BDS test	Deterministic chaos cannot be dismissed
1995	Abhyankar et al.	Hinich test, BDS test and Lypunov test	No evidence of chaos
1995	Varson and Doran	BDS test	No evidence of chaos
1997	Abhyankar et al.	Lypunov test	No evidence of chaos
1997	Serletis and Gogas	BDS test, the NEGM test, Lyapunov test	Evidence of chaos in two out of the seven series analyed
1998	Barkoulas and Travlos	BDS test	No evidence of chaos
1999	Gao and Wang	BDS test	No evidence of chaos
2000	Andreou et al.	Correlation dimensions and Lyapunov tests	Evidence of chaos in two out of four cases analysed
2001	Adrangi et al.	BDS test, correlation dimensions and Kolmogrov entropy	No evidence of chaos
2001	Gilmore	Close returns test	No evidence of chaos
2001	McKenzie	Close returns test	No evidence of chaos
2002	Urrutia	BDS test	Evidence of chaos
2002	Belaire-Franch and Contrera	Recurrence analysis	Evidence of chaos
2002	Bask	Lypunov test	No evidence of chaos
2003	Serletis and Shintani	Lyapunov test	No evidence of chaos
2004	Kyrtsou et al.	Correlation dimensions and Lyapunov tests	Evidence of noisy chaos
2004	Shintani and Linton	Lypunov exponent	No evidence of chaos
2004	Muckley	BDS test	Evidence of chaos
2005	Antoniou and Vorlow	BDS test	No evidence of chaos
2006	Urrutia	BDS test	Evidence of chaos
2007	Das and Das	Lyapunov test	Evidence of chaos
2007	Torkamani et al.	Correlation dimensions and Lyapunov tests	Evidence of chaos
2009	Liu	BDS test	Evidence of chaos
2010	Özer and Ertokatli	BDS, Hinich Bispectral, Lyapunov and NEGM tests	Evidence of chaos
2010	Adrangi et al.	Correlation dimension and BDS test	No evidence of chaos
2011	Mishra	Test of independence and Lyapunov test	Evidence of chaos
2011	Bastos and Caiado	Recurrence analysis	Evidence of chaos
2013	Diaz	BDS test, Hurts Exponent and Correlation dimension	Evidence of high-dimensional noisy chaos
2014	BenSaida	Lyapunov test	No evidence of chaos
2015	Günay	Lyapunov and BDS test	Weak evidence of chaos
2016	Sümer	Correlation dimension, BDS and Lypunov tests	Weak evidence of chaos
2017	Tsionas and Michaelides	Lyapunov test	Weak evidence of chaos
2017	Limam	The BDS and Lyapunov tests	No evidence of chaos

determined, but also the eventual source of nonlinearity because these data are usually gathered from a system whose dynamics and measurement mighty be changing over time. Moreover, the presence of measurement noise may also hinder any attempts to identify chaotic behaviour from nonlinear stochastic processes (Guégan, 2009; Granger, 2001). For this reason, the current tests used to detect chaotic structure often fail to find evidence of chaos in the data even if generated from nonlinear deterministic process.

Controversial results are also due to an inappropriate use of analytical methodologies, which are more similar to standard statistical protocols. To distinguish between chaotic and non-chaotic behaviours, all researchers, before applying chaos tests, filtered the data using both either linear and/or nonlinear models (Frank and Stengos, 1989; Blank, 1991; Cromwell and Labys, 1993; Yang and Brorsen, 1992; 1993), often doing this through the implementation of ARCH or ARFIMA and FIGARCH models (Günay, 2015). When these models fail to do grasp all of the nonlinearities existing in financial data (Hsieh, 1991; Vaidyanathan and Krebhiel 1992), chaos analysis is conducted on the residuals (Frank and Stengos, 1989; DeCoster et al., 1992; Chavas and Holt; 1991; Bask, 2002). Assuming that the residuals are filtered for linear dependence, means that any resulting dependence found out in the residuals must be nonlinear. Then, when nonlinearity is found, ARCH-type models can be applied to detect the source. If unexplained nonlinearity remains, chaos tests are applied. The application of this procedure opens the question: Are the chaotic properties of a process invariant to linear and nonlinear transformations?

It has been proved that linear and nonlinear filters can mislead potential chaotic structures (Chen 1993, Wei and Leuthold, 1998) and may affect the dimensionality of the original data (Chen, 1993, Panas and Ninni, 2000; Panas, 2002), providing a false indication of chaos. Chen (1993) showed that the correlation dimension is not invariant to the filtering through the MA (moving average model) because, in this way, the fractal structure of the dynamics is lasting.

Same conclusions both for and against chaos are reached applying a single type of chaos test. To produce convincing and reliable results, all tests for chaos have to be implemented in order to exploit their different potentials and limits. Few published papers have jointly applied the BDS test, the correlation dimension test, and the test for a positive Lyapunov exponent. Moreover, controversies are also due to the nature of the tests themselves. There may be a poor degree of robustness of such tests across variations in sample size, test methods, and data aggregation methods.

As Barnett (2006) stated "…the economics profession, to date, has provided no dependable empirical evidence of whether or not the economy itself produces chaos, and I do not expect to see any such results in the near future." Moreover, up until to now, much of research has been mainly focused on low-deterministic chaos. The failure into detecting low-dimensional chaos does not inhibit the possible existence of high-dimensional chaos in economic variables (Day, 1994). It follows that underlying nonlinear structure of the economy might be even more complex and that the related chaotic dynamics are characterized by of a higher dimensionality.

The failure in finding out convincing and reliable evidence of chaos in financial and, more in general, in economic time series, redirected research efforts in modelling nonlinearity shifting from conditional mean, properly of chaotic systems toward conditional variance, that's, ARCH-type models (Prokhorov, 2001). However, it worth noting that many researches pointed out that several are the weaknesses typical of these models, which are mainly due to the strong assumptions on which they are built (Urrutia et al., 2002; 2006; Schittenkopf et al., 2000) and the evidence of unexplained nonlinearities in the data residuals. Nevertheless, the depicted state of the art of the application of chaos theory to financial time series supports that neither naive enthusiasm to explain all kinds of unsolved time series problems by nonlinear determinism, nor is the pessimistic view that no real system is ever sufficiently deterministic and thus out of reach for analysis is justified.

4. CONCLUSIONS

Chaos in financial markets has attracted many researchers, especially when stochastic systems have failed to provide reliable forecasts, giving rise studies, showing that financial dynamics are chaotic and not stochastic. On the other hand, some others are indecisive whether these markets are stochastic or chaotic, due to the used data, and to the applied test.

According to the studies discussed in the previous sections it has be underlined that no natural deterministic explanation can justify the observed financial fluctuations produced by external shocks or by inherent randomness. In contrast to laboratory experiments, through which a large amount of data points can be easily obtained, most economic time series consist of monthly, quarterly, or annual data, with the exception of some high-frequency financial series. In fact, the analysis of financial time series has led to results more reliable than those of rising from macroeconomic series (Faggini and Parziale, 2016). This is mainly due to the fact that a great amount of the data in financial market is available, even though the literature on this topic is not free of controversial results.

Then, during the last years, the search for chaos in financial data has gradually lose its momentum, because of the lack of empirical support able to justify the presence of chaotic behaviours in these data. The current stage of chaos theory could be resumed in the words of Granger and Terasvirta (1992): "Deterministic (chaotic) models are of little relevance in economics and so we will consider only stochastic models." Jaditz and Sayers (1993) defined this issue and reviewed a huge amount of data conclude that there was no evidence for chaos, over many disaggregated time intervals, but, at the same time, the authors did not deny the indication of each sort of nonlinear dynamics. Actually it is possible conclude that there are news about chaos detecting in financial data.

Nevertheless, even if the evidences of chaos in time series data are weak, this does not imply that chaos is a not useful lens that let to approach economic activity (Brock, 1993). "The methodological obstacles in mathematics, numerical analysis, and statistics are formidable, we do not have the slightest idea of whether or not the economy exhibits chaotic nonlinear dynamics, and hence, we are not justified in excluding the possibility" (Barnett, 2006). Through the theory of chaos, it is possible to grasp the structure of unpredictability and to exhibit it in a variety of templates. The chaos theory is a revolutionary approach to understanding and forecasting the behaviour not only of financial and non-financial markets.

It is evident that until now, in economy, the chaos theory failed in providing effective responses able to overcome the mainstream approach. However, some additional response should come from the big data, which should counteract the main weakness of chaotic test tools (e.g. short data set), even if many aspects of chaos theory application to big data analytics is highly theoretical in its nature or still in their infancy. Therefore, almost all big data analysing systems currently active use the essential components of chaos theory (Gross, 2015).

In finance, because the debate still stands trying to find the answer whether stock movements are primary generated by stochastic or chaotic dynamics, the obvious goal for the near future is thus to enlarge the class of time series problems that are more efficiently solvable by the nonlinear approach. Many studies tried to identify the best model to predict future performance, but there is not clear evidence of the dominance of one approach with respect to others. If nonlinearity leads to better results with respect to the random walk hypothesis, the choice among different nonlinear approaches is not at all easy and the capability of different approaches to achieve good results is affected by the dynamics that characterizes the market. It is clear that the differences in the degree of nonlinearity identified in the financial market structure point to the impossibility of assuming that a single methodology is best without considering the specific characteristics of the market being analysed. "In fact, the different degree of nonlinearity implies a different length of the cycles that are relevant for all the forecasting methodologies and are likely to affect to a significant extent the results" (Mattarocci, 2006). In this view, chaos theory has inspired a new set of time series tools and provides a new language to formulate time series problems and to their solutions. It represents the best trade-off to establish fixed rules in order to link future dynamics to past results of a time series without imposing excessively simple assumptions but assuming that their complex dynamics may be explained if considered as a combination of more simple trends.

REFERENCES

Abhyankar, A.H., Copeland, L.S., Wong, W. (1995), Non-linear dynamics in real-time equity market indices: Evidence from the UK. The Economic Journal, 105, 864-880.

Abhyankar, A.H., Copeland, L.S., Wong, W. (1997), Uncovering nonlinear structure in real-time, stock-market indexes: The S and P 500, the DAX, the Nikkei 225, and the FTSE-100. Journal of Business and Economic Statistics, 5, 1-14.

Adrangi, B., Allender, M., Chatrath, A., Raffiee, K. (2010), Nonlinear dependencies and chaos in the bilateral exchange rate of the dollar. International Business and Economics Research Journal, 9(3), 85-96.

Adrangi, B., Chatrath, A., Dhanda, K.K., Raffiee, K. (2001), Chaos in oil prices? Evidence from futures markets. Energy Economics, 23, 405-425.

Andreou, A.S., Pavlides, G., Karytinos, A. (2000), Nonlinear time-series analysis of the Greek exchange-rate market. International Journal of Bifurcation and Chaos, 10(7), 1729-1758.

Antoniou, A., Vorlow, C.E. (2005), Price clustering and discreteness: Is there chaos behind the noise? Physica A, 348, 389-403.

Aparicio, T., Pozo, E., Saura, D. (1999), Investigating chaotic behavior in economic series: The delay time in the Grassberger-Procaccia algorithm. International Journal of Theoretical and Applied Finance, 2(4), 357-380.

Barkoulas, J., Travlos, N. (1998), Chaos in an emerging capital market? The case of the athens stock exchange. Applied Financial Economics, 8(3), 231-243.

Barnett, W.A., (2006), Comments on chaotic monetary dynamics with confidence. Journal of Macroeconomics, 28, 253-255.

Bask, M. (2002), A positive Lyapunov exponent in Swedish exchange rates. Chaos, Solitons and Fractals, 14(8), 1295-1304.

Bastos, J.A., Caiado, J. (2011), Recurrence quantification analysis of global stock markets. Physica A, 390, 1315-1325.

Bollerslev, T. (1986), Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31(3), 307-327.

Belaire-Franch, J., Contreras, D. (2002), Recurrence plots in nonlinear time series analysis: Free software. Journal of Statistical Software, 7(9), 1-18.

BenSaida, A. (2014), Noisy chaos in intraday financial data: Evidence from the American Index. Applied Mathematics and Computation, 226, 258-265.

Blank, S.C. (1991), Chaos in futures markets: A nonlinear dynamically analysis. The Journal of Futures Markets, 11, 711-728.

Brock, W.A. (1993), Pathways to randomness in the economy: Emergent nonlinearity and chaos in economics and finance. Estudios Económicos, 8(15), 3-55.

Brock, W.A., Dechert, W.D., Scheinkman, J. (1996), A test for independence based on the correlation dimension. Econometric Reviews, 15(3), 197-235.

Brock, W.A., Hsieh, D.A., Le Baron, B. (1991), Non-linear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence. Cambridge, MA: MIT Press.

Brock, W.A., Sayers, C. (1988), Is the business cycle characterized by deterministic chaos? Journal of Monetary Economics, 22(1), 71-90.

Brorsen, B.W., Yang, S. (1994), Nonlinear dynamics and the distribution of daily stock index returns. Journal of Financial Research, 17(2), 187-203.

Chavas, J., Holt, M. (1991), On nonlinear dynamic: The case of pork cycles. American Journal of Agricultural Economics, 73(3), 819-828.

Chen, P. (1993), Searching for economic chaos: A challenge to econometric practice and nonlinear tests. In: Day, R.H., Chen, P., editors. Nonlinear Dynamics and Evolutionary Economics. Oxford: Oxford University Press. p217-253.

Cromwell, J.B., Labys, W.C. (1993), Testing for Nonlinear Dynamics and Chaos in Agricultural Commodity Prices. West Virginia: Memo Institute for Labor Study, West Virginia University.

Das, A., Das, P. (2007), Chaotic analysis of the foreign exchange rates. Applied Mathematics and Computation, 185(1), 388-396.

Day, R.H. (1994), Complex Economic Dynamics: An Introduction to Dynamical Systems and Market Mechanisms, Cambridge, MA: MIT Press.

Dechert, W.D. (1996), Chaos Theory in Economics: Methods, Models and Evidence. Cheltenham, UK: Edward Elgar.

DeCoster, G.P., Labys, W.C., Mitchell, D.W. (1992), Evidence of chaos in commodity futures prices. The Journal of Future Markets, 12(3), 291-305.

Diaz, J.F.T. (2103), Evidence of noisy chaotic dynamics in the returns of four Dow Jones stock indices. Annual Review of Chaos Theory, Bifurcations and Dynamical Systems, 4, 1-15.

Ditto, W., Munakata, T. (1995), Principles and applications of chaotic systems. Communication of the ACM, 38(11), 96-102.

Engle, R. (1982), Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50(4), 987-1007.

Fama, E.F. (1965), The behavior of stock-market price. The Journal of Business, 38(1), 34-105.

Faggini, M. (2014), Chaotic time series analysis in economics: Balance and perspectives. Chaos, 24, 1-11.

Faggini, M., Parziale, A. (2016), More than 20 of chaos in economics. Mind and Society, 15(1) 53-69.

Frank, M., Stengos, T. (1988), Chaotic dynamics in economic time series. Journal of Economic Surveys, 2(2), 103-133.

Frank, M., Stengos, T. (1989), Measuring the strangeness of gold and silver rates of return. The Review of Economic Studies, 56(4), 553-567.

Gao, A.H., Wang, G.H.K. (1999), Modeling nonlinear dynamics of daily futures price changes. The Journal Futures Markets, 19(3), 325-351.

Gilmore, C.G. (1993), A new test for chaos. Journal of Economic Behavior and Organization, 22(2), 209-237.

Gilmore, C.G. (2001), An examination of nonlinear dependence in exchange rates, using recent methods from chaos theory. Global Finance Journal, 12(1), 139-151.

Granger, C.W.J. (2001), Overview of nonlinear macro-econometric empirical models. Macroeconomics Dynamics, 5(4), 466-481.

Granger, C.W.J., Terasvirta, T. (1992), Experiments in modeling nonlinear relationships between time series. In: Casdagli, M., Eubank, S., editors. Nonlinear Modeling and Forecasting, Diagnostic Testing for Nonlinearity, Chaos, and General Dependence in Time Series Data. Redwood City, California: Addison-Wesley. p189-198.

Grassberger, P., Procaccia, I. (1983a), Characterization of strange attractors. Physical Review Letters, 50(5), 346-349.

Grassberger, P., Procaccia, I. (1983b), Measuring the strangeness of strange attractors. Physica D: Nonlinear Phenomena, 9(1-2), 189-208.

Gross, T. (2015), An overwhelming amount of data: Applying chaos theory to find patterns within big data. Applied Marketing Analytics, 1(4), 377-387.

Guégan, D. (2009), Chaos in economics and finance. Annual Reviews in Control, 31(1), 89-93.

Günay, S. (2015), Chaotic structure of the BRIC countries and Turkey's

stock market. International Journal of Economics and Financial Issues, 5(2), 515-522.
Hsieh, D. (1991), Chaos and nonlinear dynamics: Applications to financial markets. The Journal of Finance, 46(5), 1839-1877.
Klioutchnikova, I., Sigovaa, M., Beizerova, N. (2017), Chaos theory in finance. Procedia Computer Science, 119, 368-375.
Kyrtsou, C., Labys, W.C., Terraza, M. (2004), Noisy chaotic dynamics in commodity markets. Empirical Economics, 29(3), 489-502.
LeBaron, B. (1994), Chaos and nonlinear forecastability in economics and finance. Philosophical Transactions of the Royal Society of London A, 348, 397-404.
Limam, M.A. (2017), Autosimilarty, long memory and chaos: Evidence from the Tunisian. Business and Management Studies, 3(2), 61-77.
Liu, L. (2009), Testing for Nonlinearity and Chaoticity in Exchange Rate Time Series, SEI. In: Mattarocci, G., editor. Market Characteristics and Chaos Dynamics in Stock Markets: An International Comparison. Available from: https://www.ssrn.com/abstract=928796; http://dx.doi.org/10.2139/ssrn.928796.
Jaditz, T., Sayers, C. (1993), Are chaotic attractors generic in economic data? International Journal of Bifurcation and Chaos, 3(3), 745-755.
Mandelbrot, B. (1963), The variation of certain speculative prices. The Journal of Business, 36(4), 394-419.
Mandelbrot, B. (1967), How long is the coast of Britain? Statistical self-similarity and fractional dimension. Science, 156(3775), 636-638.
Mattarocci, G. (2006), Market Characteristics and Chaos Dynamics in Stock Markets: An International Comparison. MPRA Paper. Available from: https://www.ssrn.com/abstract=928796.
Mayfield, E.S., Mizrach, B. (1992), On determining the dimension of real-time stock-price data. Journal of Business and Economic Statistics, 10(3), 367-374.
McKenzie, M.D. (2001), Chaotic behavior in national stock market indices. Global Finance Journal, 12(1), 35-53.
Mindlin, G.B., Hou, X.J., Solari, H.G., Gilmore, R., Tufillaro, N.B. (1990), Classification of strange attractors by integers. Physical Review Letters, 64(20), 2350-2353.
Mindlin, G.B., Gilmore, R. (1992), Topological analysis and synthesis on chaotic time series. Physica D, 58(1-4), 229-242.
Mishra, S.K. (2011), Volatility in Indian Stock Markets. Available from: http://www.ssrn.com/abstract=1788306.
Muckley, C. (2004), Empirical asset return distributions: Is chaos the culprit? Applied Economics Letters, 11(2), 81-86.
Özer, G., Ertokatli, C. (2010), Chaotic processes of common stock index returns: An empirical examination on Istanbul stock exchange (ISE) market. African Journal of Business Management, 4(6), 1140-1148.
Panas, E. (2002), Long memory and chaotic models of prices on the London Metal Exchange. Resour Policy, 27, 235-246.
Panas, U.E., Ninni, V. (2000), Are oil markets chaotic? A non-linear dynamic analysis. Energ Econ, 22, 549-568.
Pasca, L. (2015), A critical review of the main approaches on financial, market dynamics modelling. Journal of Heterodox Economics, 2(2), 152-167.
Peter, E. (1994), Fractal Market Analysis-applying Chaos Theory to Investment and Analysis. New York: John Wiley and Sons, Inc.
Potter, S. (1995), A nonlinear approach to US GNP. Journal of Applied Econometrics, 10(2), 109-125.
Prokhorov, A.B. (2001), Nonlinear Dynamics and Chaos Theory, in in Economics: A Historical Persective. Work in Progress. Available from: http://www.msu.edu/user/prohorov.
Ruelle, D. (1991), Chance and Chaos. Princeton: Princeton University Press.
Savit, R., Green, M. (1991), Time series and dependent variables. Physica D, 50(3), 95-116.
Scheinkman, J., Lebaron, B. (1989), Nonlinear dynamics and stock returns. The Journal of Business, 62(3), 311-337.
Schittenkopf, C., Dorffner, G., Dockner, E.J. (2000), On nonlinear, stochastic dynamics in economic and financial time series. Studies in Nonlinear Dynamics and Econometrics, 4(3), 101-121.
Schwartz, B., Yousefi, S. (2003), On complex behavior and exchange rate dynamics. Chaos Solitons and Fractals, 18(3), 503-523.
Serletis, A., Gogas, P. (1997), Chaos in East European black-market exchange rates. Research in Economics, 51(4), 359-385.
Serletis, A., Shintani, M. (2003), No evidence of chaos but some evidence of dependence in the U.S. stock market. Chaos Soliton Fract, 17, 449-454.
Sewell, S.P., Stansell, S.R., Lee, I., Below, S.D. (1996), Using chaos measures to examine international capital market integration. Applied Financial Economics, 6(2), 91-101.
Shintani, M., Linton, O. (2004), Nonparametric neural network estimation of Lyapunov exponents and a direct test for chaos. Journal of Econometrics, 120(1), 1-33.
Sümer, K.K. (2016), Do financial markets exhibit chaotic behavior? Evidence from BIST, Eurasian Business and Economics, 6, 1-14.
Torkamani, M.A., Mahmoodzadeh, S., Pourroostaei, S., Lucas, C. (2007), Chaos theory and alication in foreign exchange rates vs. IRR (Iranian Rial). International Journal of Human and Social Sciences, 2(7), 392-396.
Tsionas, M.G., Michaelides, P.G. (2017), Bayesian Analysis of Chaos: The Joint Return-volatility Dynamical System. MPRA Paper No. 80632.
Urrutia, J.L., Joseph, V. (2006), Empirical evidence of nonlinearity and chaos in the returns of American depository receipts. Quarterly Journal of Business and Economics, 45(1), 15-30.
Urrutia, J.L., Joseph, V., Gronewoller, P., Hoque, M. (2002), Nonlinearity and low deterministic chaotic behavior in insurance portfolio stock returns. Journal of Risk and Insurance, 69(4), 537-554.
Vaidyanathan, R., Krehbiel, T. (1992), Does the S and P 500 futures mispricing series exhibit nonlinear dependence across time? The Journal of Futures Markets, 12, 659-677.
Varson, P.L., Doran, P. (1995), The search for evidence of chaos in FTSE-100 daily returns. European Financial Management, 1(2), 201-210.
Wei, A., Leuthold, R.M. (1998), Long agricultural futures prices: ARCH, long memory, or chaos processes? University of Illinois OFOR Working Paper No. 98-3.
Wu, K., Savit, R., Brock, W. (1993), Statistical tests for deterministic effects in broad band time series. Physica D, 69(1-2), 172-188.
Yang, S.R., Brorsen, B.W. (1992), Nonlinear dynamics of daily cash prices. American Journal of Agricultural Economics, 74(3), 706-715.
Yang, S.R., Brorsen, B.W. (1993), Nonlinear dynamics of daily futures prices: Conditional heteroschedasticity or chaos? The Journal of Futures Markets, 13(2), 175-191.