International Journal of Economics and Financial Issues
Vol. 3, No. 2, 2013, pp.409-419
ISSN: 2146-4138
www.econjournals.com

Can Child-Care Support Policies Halt Decreasing Fertility?[†]

Masaya Yasuoka
Faculty of Economics and Business Administration, The University of Kitakyushu,
4-2-1 Kitagata, Kokuraminami, Kitakyushu, 802-8577, Japan.
Tel.: +81-93-964-4318. Email: yasuoka@kitakyu-u.ac.jp

ABSTRACT: Some earlier papers examine whether child allowances can raise fertility or not in an endogenous fertility model with a defined contribution pension system. They derive that a child allowance can raise fertility. This paper is aimed at deriving the level of child allowances or education subsidies to make the pension system sustainable. A child allowance can raise fertility instantaneously. However, in the long run, fertility might continue decreasing and the pension system might not be sustainable if less child allowance is provided. In a defined benefit system, tax burdens for pension benefits are heavy in an aging society with fewer children. A heavy tax burden reduces the household income and then decreases fertility. Therefore, child allowances must be provided to halt decreasing fertility in the long run. Nevertheless, given parametric conditions, education subsidy of more than a certain level can not halt the decrease of fertility in the long run.

Keywords: Child allowances; Education subsidy; Defined benefit; Endogenous fertility
JEL Classifications: G23; H55; J13

1. Introduction

This paper presents examination of whether child care support policies can raise fertility and education investment for children or not in an endogenous fertility model with a pay-as-you-go pension (unfunded pension), which is fixed benefit (defined benefit (DB)). A pay-as-you-go pension reduces fertility because older people do not need a gift from their children, as reported by Zhang (1995), Zhang and Zhang (1998), Wigger (1999), Oshio and Yasuoka (2009), and others. Child-care support policies to halt the decrease in fertility must be provided to sustain the pension system in an aging society with fewer children. Oshio (2001), van Groezen et al., (2003), van Groezen and Meijdam (2008), and Mochida (2009) derived that child allowances can raise fertility in a defined contribution pension (defined contribution (DC)). However, fertility can not always be raised by child allowances in a closed economy because of the decrease in income per capita, as presented by Fanti and Gori (2009).[1]

Child allowances are a subsidy that is given based on the number of children. We can consider an education subsidy as another child care support policy which is a subsidy for the quality of children. Zhang (1997) and Zhang and Casagrande (1998) examined the effects of child care policies on fertility and human capital accumulation and derived that child allowances can increase fertility, but decreased human capital of children and education subsidies can increase children human capital but decrease fertility through substitution between the quality and quantity of children.[2] However, an education subsidy does not always reduce fertility. Yasuoka and Miyake (2008) derived that an

[†] I would like to thank Makoto Hirazawa, Kei Hosoya, Koji Kitaura, Tomoya Sakagami, Yurika Shiozu and seminar participants at the 2012 Spring Meeting of Japan Association for Applied Economics and 2012 Japanese Economic Association Autumn Meeting for their helpful comments and suggestions. However, any remaining errors are mine. Research for this paper was financially supported by a Grant-in-Aid for Scientific Research (No.23730283).

[1] Some earlier papers have described that an increase in income per capita decreases fertility. Galor and Weil (1996) set a model in which the child care time to have children is necessary and derived that an increase in opportunity cost to stop working caused by an increase in income per capita reduces fertility.

[2] de la Croix and Doepke (2003) set a model with quality and quantity of children. An increase in human capital accumulation reduces fertility because of an increase in opportunity costs of having children.

education subsidy raises both human capital accumulation and fertility in a model with a pay-as-you-go pension. Fanti and Gori (2010, 2011) considered a model with public education and derived a result in which an increase in public education raises fertility.[3]

Oshio (2001), van Groezen, et al., (2003), Yasuoka (2006), van Groezen and Meijdam (2008), Mochida (2009), and Yasuoka and Goto (2011) examine the effects of child allowances on fertility in a model with a DC pension. Some earlier papers present examination of the effect of child allowances on fertility. Oshio and Yasuoka (2009) set an endogenous fertility with a DB pension and derived the level of child allowances to sustain the DB pension. Child allowances can raise fertility in the short run. However, if fertility does not increase to any great degree, then child allowances can not stop decreasing fertility because of a tax burden to provide pension benefits. This is not derived in the model with a DC pension.[4]

This paper presents consideration of an endogenous fertility model with DB pension in a small open economy and aims to derive the level of child allowances examined by Oshio and Yasuoka (2009) and the level of an education subsidy to sustain a DB pension. The conclusions presented in this paper are shown as follows. Under some parametric conditions, two steady state equilibria exist: one for low fertility and the other for high fertility. Depending on an initial condition, fertility converges to zero or a high fertility level. However, without steady state equilibrium, fertility continues decreasing to zero. Child allowances and education subsidies are necessary to prevent continued decreasing. As shown in the results presented in this paper, more than a certain level of child allowances can halt the decrease in fertility and can sustain a DB pension. However, a certain level of education subsidy can not always bring about a steady state with constant fertility to sustain a DB pension. Given some parametric conditions, fertility converges to zero in the long run because of the great amount of the education subsidy.

In Japan, the ratio of older people to the total population is 23.1%. The total fertility rate was 1.39 in 2010.[5] The contribution rate for pensions is increasing, creating a heavy burden for households. Pension reform in 2004 in Japan was aimed at regulating pension benefits by the government to avoid imposing a heavy burden on younger people (Macroeconomic Slide System). However, reforms have still not been conducted. Current circumstances are not sustainable because pension benefits are not adequately financed by payments from younger people if this burden per household is heavy. Therefore, it is necessary that an increase in the future working population and income per capita be made to decrease the tax burden of households and to create a pension system that is sustainable. Child-care support policies must be provided.[6] This paper presents examination of the level of child allowances and education subsidies to make the pension system sustainable. This analysis is important.

This paper consists of the following. Section 2 sets an endogenous fertility model with the DB pension model. Section 3 derives the equilibrium and examines the effects of child care support policies on fertility and human capital accumulation. Section 4 concludes with a presentation of the results obtained in this paper.

2. Model

This model economy consists of a two-period (young and old) overlapping generations model and assumes a small open economy. An interest rate 1+r and wage rate w is given exogenously. In the following subsection, we explain each agent.

---

[3] Public education is examined by Glomm and Ravikumar (1992) and others.

[4] Lin and Tian (2003) describe an endogenous fertility model with a DB pension and examined whether tax reform financed by consumption tax can raise social welfare or not. Borgmann (2005) considered an exogenous fertility model with uncertainty and whether social welfare in DB pension is greater than that in a DC pension.

[5] Data: Demographic Statistics (Ministry of Health, Labour and Welfare, Japan), Population Estimates (Ministry of Internal Affairs and Communications, Japan), White Paper on Birthrate-Declining Society (Cabinet Office, Japan)

[6] In Japan, child-care support policies are insufficiently provided. Fiscal support for families, which is measured by government expenditure for child care to GDP, is 0.79% at 2007 (Data: Cabinet Office, Government of Japan (2011)). This level is less than that in France (3.0%), Sweden (3.35%), and other European countries. The ratio of public education expenditure to GDP is 3.3% in 2007, which is less than that of the OECD average (4.8%) (Data: OECD (2010) Education at a Glance 2010).

2.1 Households

Each household lives in three periods--childhood, young, and old periods--and supplies labor to earn an income during the young period. Young people supply labor inelastically for consumption during the young period and use savings to pay for consumption during the old period in addition to caring for children and providing education for their children. A government provides not only a pension system that gives older people a fixed benefit but also a child allowance and education subsidy for younger people. Consequently, a household's lifetime budget constraint is given as[7]

$$c_{1t} + \frac{c_{2t+1}}{1+r} + (z-q)n_t + (1-i)e_t n_t = (1-\tau_t - \theta_t)wh_t + \frac{p}{1+r}. \tag{1}$$

Therein, n_t and e_t respectively represent the number of children and the quality of children (education investment). Necessary goods to bring up a child are represented as z. Both q and i denote child allowances (subsidy for the quantity of children) and education subsidy (subsidy for the quality of children). In addition, c_{1t} and c_{2t+1} respectively denote consumption during the young period and that during the old period. h_t denote the human capital stock. Younger people face labor income taxation (tax rate or contribution rate τ_t for pension benefit and a tax rate θ_t for child allowances and education subsidies). Older people receive pension benefit p. Furthermore, t signifies the period. A household's utility function is assumed as[8]

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1-\alpha-\beta)\ln n_t h_{t+1}, \quad 0 < \alpha, \beta < 1, \alpha + \beta < 1. \tag{2}$$

Human capital in the subsequent period h_{t+1} is accumulated by education investment e_t and the parents' human capital h_t based on the following equation.[9]

$$h_{t+1} = A e_t^{\varepsilon} h_t^{1-\varepsilon}, \quad 0 < A, 0 < \varepsilon < 1. \tag{3}$$

Under budget constraints (1) and (3), allocations of c_{1t}, c_{2t+1}, n_t, and e_t to maximize utility (3) shown as

$$c_{1t} = \alpha\left((1-\tau_t-\theta_t)wh_t + \frac{p}{1+r}\right), \tag{4}$$

$$c_{2t+1} = (1+r)\beta\left((1-\tau_t-\theta_t)wh_t + \frac{p}{1+r}\right), \tag{5}$$

$$n_t = \frac{(1-\varepsilon)(1-\alpha-\beta)\left((1-\tau_t-\theta_t)wh_t + \frac{p}{1+r}\right)}{z-q}, \tag{6}$$

$$e_t = \frac{\varepsilon}{1-\varepsilon}\frac{z-q}{1-i}. \tag{7}$$

Child allowance q raises fertility n_t and education subsidy i pulls up education investment e_t.

2.2 Government

The government executes two policies: one for the pension and one for child care support policies (child allowances and education subsidy). A payroll tax rate θ_t is levied on young people. This tax revenue is used as child allowances and education subsidy. With a balanced budget, we obtain the following equation.

$$qn_t + ie_t n_t = \theta_t wh_t. \tag{8}$$

Moreover, the government collects payroll tax revenue at tax rate τ_t from younger people to give a pension benefit for older people. We consider a defined benefit (DB) by which pension benefit p is fixed and tax rate τ_t is adjusted to hold the balanced budget. Then, the government budget constraint is shown as

---

7 Becker and Barro (1988) and Barro and Becker (1989) assume a utility function reflecting parents' concern about their children's welfare. However, Eckstein and Wolpin (1985) assume a utility function that reflects parents' concerns when having children.

8 This utility function contains n_t and h_{t+1}, as assumed by de la Croix and Doepke (2003) and others.

9 Zhang (1997) and Zhang and Casagrande (1998) assume the same human capital accumulation equation.

$$\tau_t = \frac{p}{n_{t-1} w h_t}. \tag{9}$$

The tax burden per capita τ_t becomes low if fertility n_{t-1} or human capital h_t is large. Therefore, fertility or human capital raised by child care support policies contributes to a decrease in the tax burden.

3. Equilibrium

In this section, we consider equilibria of two types: one for the equilibrium with child allowances and one for that with an education subsidy. First, we consider the equilibrium with child allowances.

3.1 Child Allowances

Considering i=0, (8) reduces to $qn_t = \theta_t w h_t$. Substituting this equation and (9) into (6), we obtain the following equation.

$$n_t = \frac{(1-\varepsilon)(1-\alpha-\beta)}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\left(wh_t + \frac{p}{1+r} - \frac{p}{n_{t-1}}\right) \tag{10}$$

By substituting (7) into (3), human capital in t+1 period is shown as

$$h_{t+1} = A\left(\frac{\varepsilon(z-q)}{1-\varepsilon}\right)^{\varepsilon} h_t^{1-\varepsilon}. \tag{11}$$

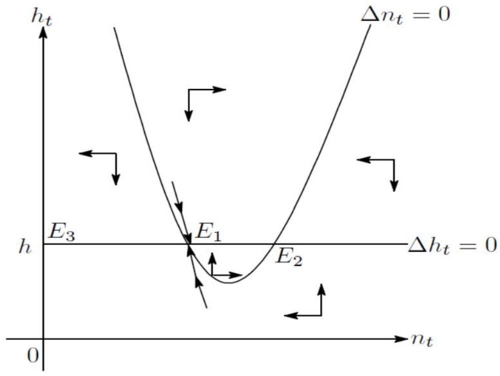
Defining $\Delta n_t \equiv n_{t+1} - n_t$ and $\Delta h_t \equiv h_{t+1} - h_t$, we obtain the following equations.

$$\Delta n_t = \frac{(1-\varepsilon)(1-\alpha-\beta)}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\left(wA\left(\frac{\varepsilon(z-q)}{1-\varepsilon}\right)^{\varepsilon} h_t^{1-\varepsilon} + \frac{p}{1+r} - \frac{p}{n_t}\right) - n_t, \tag{12}$$

$$\Delta h_t = A\left(\frac{\varepsilon(z-q)}{1-\varepsilon}\right)^{\varepsilon} h_t^{1-\varepsilon} - h_t \tag{13}$$

The loci of $\Delta n_t = 0$ and $\Delta h_t = 0$ are depicted in Fig. 1.[10]

Figure 1-1. Dynamics of n_t.



Based on parametric conditions, the dynamics of two types of n_t and h_t are shown. Figure 1-1 shows the case with two steady state equilibria: E_1 and E_2. The former, E_1, is a saddle-point stable steady state equilibrium; E_2 is a sink. Depending on initial conditions h_0 and n_0, the dynamics of n_t and h_t converge to E_2 which brings about higher fertility than that at E_1 or E_3, which brings no fertility. Figure 1-2 shows the case of no steady state equilibrium. Then, h_t and n_t converge to E_3 for any h_0 and n_0.

[10] See for Appendix for detail proof about the locus of $\Delta n_t = 0$.

Fertility n and the human capital stock h in the steady state are shown as follows if this economy has two steady state equilibria E_1 and E_2, as

$$n=\frac{1}{2}\left(\frac{(1-\varepsilon)(1-\alpha-\beta)}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\left(wh+\frac{p}{1+r}\right)-\sqrt{D_q}\right) \quad \text{for } E_1, \qquad (14)$$

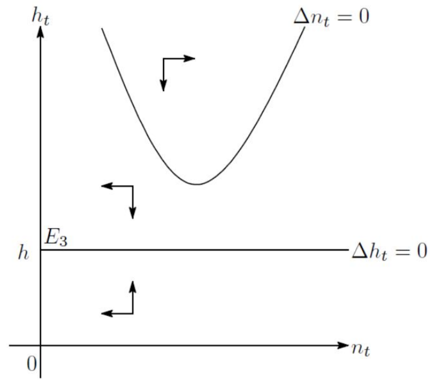
$$n=\frac{1}{2}\left(\frac{(1-\varepsilon)(1-\alpha-\beta)}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\left(wh+\frac{p}{1+r}\right)+\sqrt{D_q}\right) \quad \text{for } E_2, \qquad (15)$$

$$h=\frac{A^{\frac{1}{\varepsilon}}\varepsilon(z-q)}{1-\varepsilon}, \qquad (16)$$

where $D_q \equiv \left(\frac{(1-\varepsilon)(1-\alpha-\beta)\left(wh+\frac{p}{1+r}\right)}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\right)^2-\frac{4(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}.$

With $D_q>0$, we obtain the two steady state equilibria, depicted in Fig. 1-1. However, if $D_q<0$, then no steady state exists, as depicted in Fig. 1-2.

Figure 1-2. Dynamics of n_t (no steady state).



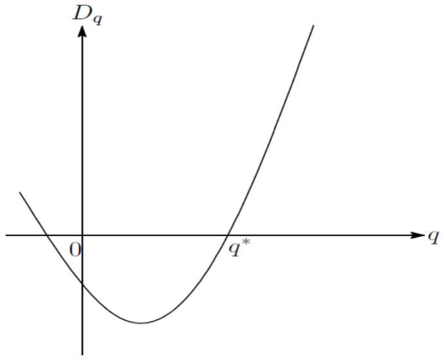
Now, we assume the economy with $D_q<0$ at $q=0$.[11] That is, fertility converges to zero in the long run. However, by virtue of child allowances, D_q changes from a negative value to a positive one and then two steady state equilibrium might be brought about. Child allowances can bring about the steady state equilibrium which does not continue decreasing fertility if $D_q>0$. The child allowances which hold $D_q>0$ are depicted in Fig. 2.[12]

If a greater than q^* child allowance is provided, then the two steady state equilibria are brought about even if no steady state equilibrium existed before provision of child allowances. Child allowances can not bring about the steady state equilibrium because n_t given by (10) must not be negative if $q^*>\frac{z}{1-(1-\varepsilon)(1-\alpha-\beta)}$. Then, we establish the following proposition.

[11] $D_q<0$ at $q=0$ is $(1-\varepsilon)(1-\alpha-\beta)\left(\frac{wA^{\frac{1}{\varepsilon}}\varepsilon z}{1-\varepsilon}+\frac{p}{1+r}\right)^2<4pz$.

[12] $D_q<0$ at $q=0$. Therefore, q exists to hold $D_q=0$ as shown in Fig. 2.

Figure 2. Child allowance q to hold D_q>0.



Proposition 1

We assume no steady state economy without child allowances. Child allowances can bring about the steady state and stop decreasing fertility if q>q*, where $q^* < \frac{z}{1-(1-\varepsilon)(1-\alpha-\beta)}$.[13]

Proposition 1 shows that if child allowances that amount to more than a certain level are provided, then the steady state equilibrium is brought about, as shown in Fig. 1-1. Eq. (10) shows that child allowances can raise fertility n_t for any n_{t-1}. This result is the same as that reported by van Groezen, Leers and Meijdam (2003). In the model with defined contribution examined by van Groezen et al., (2003), child allowances can raise fertility both in the short term and in the long run with a DC pension. However, in the model with a DB pension examined in this paper, fertility might not increase in the long run. Without a steady state equilibrium, even if child allowances can raise fertility in the short run, no steady state equilibrium makes fertility continue decreasing. If the pension system is managed as a DC pension, this result is not derived. Even if child allowances can raise fertility in the short run, the tax burden is heavy when fertility is low. Then a household's income decreases. A decrease in the household's income reduces fertility because the household decreases the payment for child care. Finally, fertility continues decreasing and converges to zero in the long run because of the heavy tax burden. Therefore, the child allowances must be provided at more than a certain level because of the cessation of a decrease in the long run. Fanti and Gori (2009, 2011) reported that child allowances can not raise fertility with a DC pension in the long run. However, the reason differs from that described in this paper. Fanti and Gori (2009) explained that child allowances reduce the per-capita capital stock and that household incomes decrease. Fanti and Gori (2010) explained that an increase in fertility created by child allowances reduces the quality of public education. Then human capital accumulation is prevented. Finally household incomes decrease.

3.2 Education Subsidy

Considering q=0, (8) reduces to $ie_t n_t = \theta_t w h_t$. Substituting this equation and (9) into (6), we obtain the following equation:

$$n_t = \frac{(1-\varepsilon)(1-\alpha-\beta)\left(wh_t - \frac{p}{n_{t-1}} + \frac{p}{1+r}\right)}{\left(1+\frac{\varepsilon i(1-\alpha-\beta)}{1-i}\right)z}. \quad (17)$$

By substituting (7) into (3), human capital in t+1 period is shown as

[13] If the preference for children is small, that is $1-\alpha-\beta$ is small, then $\frac{z}{1-(1-\varepsilon)(1-\alpha-\beta)}$ is small. Therefore, q* might be greater than $\frac{z}{1-(1-\varepsilon)(1-\alpha-\beta)}$ when $1-\alpha-\beta$ is small: it is difficult to bring about the steady state equilibrium with child allowances.

$$h_{t+1} = A\left(\frac{\varepsilon z}{(1-\varepsilon)(1-i)}\right)^{\varepsilon} h_t^{1-\varepsilon}. \tag{18}$$

Then, $\Delta n_t = 0$ and $\Delta h_t = 0$ are shown by the following equations.

$$\Delta n_t = \frac{(1-\varepsilon)(1-\alpha-\beta)\left(wA\left(\frac{\varepsilon z}{(1-\varepsilon)(1-i)}\right)^{\varepsilon} h_t^{1-\varepsilon} - \frac{p}{n_{t-1}} + \frac{p}{1+r}\right)}{\left(1+\frac{\varepsilon i(1-\alpha-\beta)}{1-i}\right)z} - n_t, \tag{19}$$

$$\Delta h_t = A\left(\frac{\varepsilon z}{(1-\varepsilon)(1-i)}\right)^{\varepsilon} h_t^{1-\varepsilon} - h_t. \tag{20}$$

As shown by the case of child allowances, the loci of $\Delta n_t = 0$ and $\Delta h_t = 0$ are depicted in Fig. 1.

The fertility n and the human capital stock h in the steady state are shown as follows if this economy has two steady state equilibria E_1 and E_2.

$$n = \frac{(1-\varepsilon)(1-\alpha-\beta)\left(\frac{A^{\frac{1}{\varepsilon}}\varepsilon z w}{(1-\varepsilon)(1-i)} + \frac{p}{1+r}\right) - \sqrt{D_i}}{2z\left(1+\frac{i\varepsilon(1-\alpha-\beta)}{1-i}\right)} \quad \text{for } E_1, \tag{21}$$

$$n = \frac{(1-\varepsilon)(1-\alpha-\beta)\left(\frac{A^{\frac{1}{\varepsilon}}\varepsilon z w}{(1-\varepsilon)(1-i)} + \frac{p}{1+r}\right) + \sqrt{D_i}}{2z\left(1+\frac{i\varepsilon(1-\alpha-\beta)}{1-i}\right)} \quad \text{for } E_2, \tag{22}$$

$$h = \frac{A^{\frac{1}{\varepsilon}}\varepsilon z}{(1-\varepsilon)(1-i)}, \tag{23}$$

where

$$D_i \equiv \left[(1-\varepsilon)(1-\alpha-\beta)\left(\frac{A^{\frac{1}{\varepsilon}}\varepsilon z w}{(1-\varepsilon)(1-i)} + \frac{p}{1+r}\right)\right]^2 - 4(1-\varepsilon)(1-\alpha-\beta)pz\left(1+\frac{i\varepsilon(1-\alpha-\beta)}{1-i}\right).$$

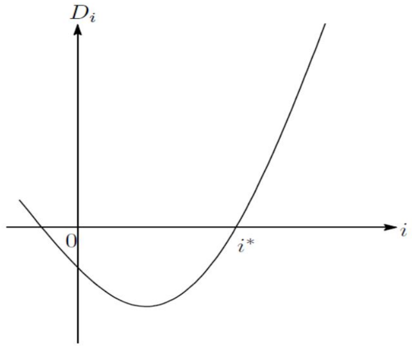
With $D_i>0$, we obtain the two steady state equilibria depicted in Fig. 1-1. However, if $D_i<0$, then no steady state exists, as depicted in Fig. 1-2. Now, we assume the economy with $D_i<0$ at $i=0$.[14] That is, we consider the economy without education subsidy where fertility converges to zero in the long run. The condition to have two steady state equilibria is shown as

[14] This condition is the same with $D_q<0$ at $q=0$.

$$\left[\left(\frac{p}{1+r}\right)^2-\frac{4pz(1-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}\right]i^2$$
$$-2\left[\left(\frac{p}{1+r}\right)^2+\frac{pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}-\frac{4pz(2-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}\right]i \tag{24}$$
$$+\left(\frac{p}{1+r}\right)^2+\frac{2pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}+\left(\frac{A^{\frac{1}{\varepsilon}}\varepsilon zw}{1-\varepsilon}\right)^2-\frac{4pz}{(1-\varepsilon)(1-\alpha-\beta)}>0.$$

We define $Y\equiv\left(\frac{p}{1+r}\right)^2-\frac{4pz(1-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}$. Then, if Y>0, then the education subsidy i to be $D_i>0$ exists, as shown in Fig. 3-1.

Figure 3-1. Education subsidy to hold $D_i>0$ (Y>0).



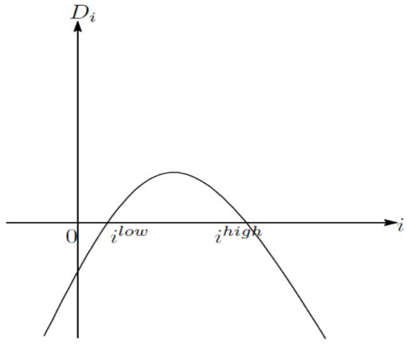
Two steady state equilibria which stop decreasing fertility if an education subsidy is provided by more than i*<i. If 1<i*, then the education subsidy brings about no steady state equilibrium because i must be less than one. We define the following equation.

$$Z\equiv 4\left[\left(\frac{p}{1+r}\right)^2+\frac{pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}-\frac{2pz(2-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}\right]^2$$
$$-4Y\left[\left(\frac{p}{1+r}\right)^2+\frac{2pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}+\left(\frac{A^{\frac{1}{\varepsilon}}\varepsilon zw}{1-\varepsilon}\right)^2-\frac{4pz}{(1-\varepsilon)(1-\alpha-\beta)}\right]. \tag{25}$$

We consider the case of Y<0. If Z>0 and $\left(\frac{p}{1+r}\right)^2+\frac{pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}-\frac{2pz(2-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}<0$ hold, then an education subsidy set as $i^{low}<i<i^{high}$ brings about two steady state equilibria, as shown in Fig. 3-2.[15] However, if Z<0 or Z>0 and $-2\left[\left(\frac{p}{1+r}\right)^2+\frac{pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}-\frac{2pz(2-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}\right]<0$, then the education subsidy can not bring about the steady state. Consequently, the following proposition is established.

[15] No education subsidy giving two steady state equilibria exists if $1<i^{low}$.

Figure 3-2. Education subsidy to hold D_i>0 (Y<0).



Proposition 2

We assume no steady state economy. For Y>0, an education subsidy can bring about the steady state and stop decreasing fertility if education subsidy i*<i is provided. in the case of Y<0, the education subsidy provided by $i^{low}<i<i^{high}$ can bring about two steady state equilibria as long as Z>0 and $\left(\frac{p}{1+r}\right)^2+\frac{pA^{\frac{1}{\varepsilon}}\varepsilon zw}{(1-\varepsilon)(1+r)}-\frac{2pz(2-\varepsilon(1-\alpha-\beta))}{(1-\varepsilon)(1-\alpha-\beta)}<0$ hold.

Being different from the analysis of child allowances, the analysis of education subsidy presents some interesting results. An education subsidy provided at more than i* can bring about the steady state equilibrium if Y>0. This case is the same as that of the result for child allowances. However, if Y<0, then an education subsidy provided between i^{low} and i^{high} can bring about the steady state equilibrium: a high amount of education subsidy can not bring about the steady state equilibrium because fertility is decreased greatly by a high amount of education subsidy, as shown by (17). The sign of Y becomes negative if the child care cost z is large: in the case of high child care cost, the education subsidy must not be provided beyond the certain level and child allowances should be provided to bring about the steady state equilibrium. Yasuoka and Miyake (2008) and Fanti and Gori (2010) derived that an education subsidy raises fertility because of an increase in human capital (therefore, household income) in the long run. However, an education subsidy higher than a certain level eliminates the steady state equilibrium and fertility converges to zero in this model.

4. Conclusions

This paper presents an examination of whether child care support policies can stop decreasing fertility or not in the long run by considering an endogenous fertility model with a pay-as-you-go pension. Child allowances and education subsidy can raise fertility in the long run, as derived in earlier studies, if the pension system is a defined contribution (DC) pension. However, if the pension system is a defined benefit (DB) pension, although child allowances and education subsidy can raise fertility instantaneously, child allowances and education subsidies can not always halt decreasing fertility in the long run. In a DB pension scheme, the tax burden for pension benefits is too heavy in an aging society with fewer children. Even if fertility is raised by child allowances in the short run, fertility continues decreasing in the long run because child allowances and education subsidies can not halt a continued increase of the tax burden per capita as long as pension benefits are fixed. Households reduce the number of children because of a lack of disposable income. The result related to education subsidies is noteworthy. More than a certain level of education subsidy can not halt decreasing fertility, because of the substitution between quality and quantity of children, if child care cost is high.

References

Becker G.S., Barro, R.J. (1988), "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 103, 1-25.

Barro, R.J., Becker, G.S. (1989), "Fertility Choice in a Model of Economic Growth," *Econometrica*, 57(2), 481-501.

Borgmann, C. (2005), 'Social Security, Demographics, and Risk,' Springer Verlag, Berlin Heidelberg.
de la Croix, D., Doepke, M. (2003), "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review*, 93(4), 1091-1113.
Eckstein, Z., Wolpin, K.I. (1985), "Endogenous Fertility and Optimal Population Size," *Journal of Public Economics*, 27, 93-106.
Fanti, L., Gori, L. (2009), "Population and Neoclassical Economic Growth: a New Child Policy Perspective," *Economics Letters*, 104, 27-30.
Fanti, L., Gori, L. (2010), "Public Education, Fertility Incentives, Neoclassical Economic Growth and Welfare," *Bulletin of Economic Research*, 62(1), 59-77.
Fanti, L., Gori, L. (2011), "Child Policy Ineffectiveness in an Overlapping Generations Small open Economy with Human Capital Accumulation and Public Education," *Economic Modelling*, 28, 404-409.
Galor, O., Weil, N. (1996), "The Gender Gap, Fertility, and Growth," *American Economic Review*, 86(3), 374-387.
Glomm, G., Ravikumar, B. (1992), "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, 100(4), 818-834.
Lin, S., Tian, X. (2003), "Population Growth and Social Security Financing," *Journal of Population Economics*, 16, 91-110.
Mochida, M. (2009), "Child-Allowances, Fertility, and Uncertain Lifetime," *Economics Bulletin*, 29(4), 2712-2721.
Oshio, T. (2001), "Child-Care Reform in Pension System and the Fertility Rate (*in Japanese*)," *Quarterly of Social Security Research*, 36(4), 535-546.
Oshio, T., Yasuoka, M. (2009), "Maximum Size of Social Security in a Model of Endogenous Fertility," *Economics Bulletin*, Vol. 29-2, pp. 656-666.
Van Groezen, B., Leers, van T., Meijdam, L. (2003), "Social Security and Endogenous Fertility: Pension and Child Allowances as Siamese Twins," *Journal of Public Economics*, 87, 233-251.
Van Groezen, B., Meijdam, L. (2008), "Growing Old and Staying Young: Population Policy in an Ageing Closed Economy," *Journal of Population Economics*, 21, 573-588.
Wigger, B. (1999), "Pay-As-You-Go Financed Public Pensions in a Model of Endogenous Growth and Fertility," *Journal of Population Economics*, 12(4), 625-640.
Yasuoka, M. (2006), "The Relationship between Fertility Rate and Tax Policy (*in Japanese*)," *Quarterly of Social Security Research*, 42(1), 80-90.
Yasuoka, M., Miyake, A. (2008), "Fertility Rate and Childcare policies in a Pension System", *Proceeding of the 64th Congress of the International Institute of Public Finance*.
Yasuoka, M., Goto, N. (2011), "Pension and Child Care Policies with Endogenous Fertility," *Economic Modelling*, 28, 2478-2482.
Zhang, J. (1995), "Social Security and Endogenous Growth," *Journal of Public Economics*, 58, 185-213.
Zhang, J. (1997), "Fertility, Growth and Public Investments in Children," *Canadian Journal of Economics*, 30(4), 835-843.
Zhang, J., Casagrande, R. (1998), "Fertility, Growth, and Flat-rate Taxation for Education Subsidies," *Economics Letters*, 60, 209-216.
Zhang, J., Zhang, J. (1998), "Social Security, Intergenerational Transfers, and Endogenous Growth," *Canadian Journal of Economics*, 31(5), 1225-1241.

Appendix

Locus of $\Delta n_t = 0$

Considering $\Delta n_t = 0$ and differentiating of $\Delta n_t = 0$ at n_t and h_t, we obtain the following equation.

$$\frac{dh_t}{dn_t} = \frac{1 - \dfrac{1}{n_t^2} \dfrac{(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}}{\dfrac{(1-\varepsilon)^2(1-\alpha-\beta)wA}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}\left(\dfrac{\varepsilon(z-q)}{1-\varepsilon}\right)^{\varepsilon} h_t^{-\varepsilon}}. \tag{26}$$

We define the numerator of (26) as $X(n_t)$. Therefore, we obtain

$$X(n_t) = \frac{1}{n_t^2}\left(n_t + \sqrt{\frac{(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}}\right)\left(n_t - \sqrt{\frac{(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}}\right) \tag{27}$$

Therefore, if $n_t > \sqrt{\dfrac{(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}}$, then $X(n_t)>0$, that is $\dfrac{dh_t}{dn_t} > 0$. That is $\dfrac{dh_t}{dn_t} < 0$ if $n_t < \sqrt{\dfrac{(1-\varepsilon)(1-\alpha-\beta)p}{z-(1-(1-\varepsilon)(1-\alpha-\beta))q}}$, then $X(n_t)<0$.